Supplemental file of the paper titled "Surveillance of Cardiovascular Diseases Using A Multivariate Dynamic Screening System"

To save some space in the paper of the above title, some technical details and numerical results are presented in this supplemental file. This file has two sections. In Section 1, the proofs of the four theorems described in Section 3 of the paper are provided. In Section 2, some extra numerical results about the MDySS method discussed in the paper are presented.

1 Proofs of the Four Theorems in Section 3 of the Paper

1.1 Proof of Theorem 3.1

By equation (3) in the paper, we have

$$\mathrm{Cov}(\widehat{\boldsymbol{\beta}}|\mathcal{F}_m) = A_m^{-1} B_m A_m^{-1}, \tag{S.1}$$

where $\mathcal{F}_m$ is the σ-field generated by $\{t_{ij},\ j=1,2,\ldots,J_i, i=1,2,\ldots,m\}$,

$$\begin{aligned} A_m &= \sum_{i=1}^m (I_{q\times q}\otimes X_i)' K_i^{\frac{1}{2}} (\tilde{I}_i V_i \tilde{I}_i)^{-1} K_i^{\frac{1}{2}} (I_{q\times q}\otimes X_i), \\ B_m &= \sum_{i=1}^m (I_{q\times q}\otimes X_i)' K_i^{\frac{1}{2}} (\tilde{I}_i V_i \tilde{I}_i)^{-1} K_i^{\frac{1}{2}} V_i K_i^{\frac{1}{2}} (\tilde{I}_i V_i \tilde{I}_i)^{-1} K_i^{\frac{1}{2}} (I_{q\times q}\otimes X_i), \end{aligned}$$

$\widetilde{I}_i = \mathrm{diag}\{\widetilde{I}_{i1},\ldots,\widetilde{I}_{iJ_i}\}$, and $\widetilde{I}_{ij} = \mathrm{diag}\{I(|t_{ij}-t| < h_1),\ldots,I(|t_{ij}-t| < h_q)\}$, for $j=1,2,\ldots,J_i$ and $i=1,2,\ldots,m$. Without loss of generality, assume that $h_1 \leq$

$h_2 \leq \cdots \leq h_q$. Let $\widetilde{H} = \text{diag}\{1, h_q, ..., h_q^p\}$, $a_{r+1,l+1}^{(sk)}$ denote the $((s-1)(p+1)+r+1, (k-1)(p+1)+l+1)$-th element of A_m, and $\boldsymbol{C}_{ijsr}(t)$ be a q-dimensional vector with the sth element equal to $(t_{ij}-t)^r K_{h_s}^{1/2}(t_{ij}-t)$ and other elements to be 0, for $s = 1, 2, \ldots, q$. Then, we have

$$
\begin{aligned}
& E(a_{r+1,l+1}^{(sk)}) \\
= & \sum_{i=1}^{m}\sum_{j=1}^{J_i} E\left\{(0,...,0,\boldsymbol{C}_{ijsr}'(t),0,...,0)(\tilde{I}_i V_i \tilde{I}_i)^{-1}(0,...,0,\boldsymbol{C}_{ijkl}'(t),0,...,0)'\right\} \\
= & \sum_{i=1}^{m} J_i \int \{\boldsymbol{C}_{i1sr}'(t)(\tilde{I}_{i1}\Sigma(t_{i1},t_{i1})\tilde{I}_{i1})^{-1}\boldsymbol{C}_{i1kl}(t)\} f(t_{i1})\, dt_{i1} \\
= & \sum_{i=1}^{m} J_i \int (t_{i1}-t)^{r+l} K_{h_s}^{\frac{1}{2}}(t_{i1}-t) K_{h_k}^{\frac{1}{2}}(t_{i1}-t) f(t_{i1}) e_s'(\tilde{I}_{i1}\Sigma(t_{i1},t_{i1})\tilde{I}_{i1})^{-1} e_k\, dt_{i1} \\
= & \sum_{i=1}^{m} J_i \sum_{v=1}^{\min(s,k)} \left[\int_{\{h_{v-1}<|t_{i1}-t|\leq h_v\}} (t_{i1}-t)^{r+l} K_{h_s}^{\frac{1}{2}}(t_{i1}-t) K_{h_k}^{\frac{1}{2}}(t_{i1}-t) f(t_{i1}) \times \right. \\
& \left. \boldsymbol{e}_s'(\tilde{I}_{i1}\Sigma(t_{i1},t_{i1})\tilde{I}_{i1})^{-1}\boldsymbol{e}_k\, dt_{i1}\right] \\
= & \sum_{i=1}^{m} J_i f(t) \sum_{v=1}^{\min(s,k)} \sigma_v^{sk}(t) \left[\int_{\{h_{v-1}<|t_{i1}-t|\leq h_v\}} (t_{i1}-t)^{r+l} K_{h_s}^{\frac{1}{2}}(t_{i1}-t) K_{h_k}^{\frac{1}{2}}(t_{i1}-t)\, dt_{i1}\right]\{1+o(1)\} \\
= & J_S h_q^{r+l+1} f(t) \sum_{v=1}^{\min(s,k)} \sigma_v^{sk}(t) \left[\int_{\{h_{v-1}/h_q<|z|\leq h_v/h_q\}} z^{r+l} K_{h_s}^{\frac{1}{2}}(h_q z) K_{h_k}^{\frac{1}{2}}(h_q z)\, dz\right]\{1+o(1)\} \\
= & J_S h_q^{r+l} \xi_{skrl}(t)\{1+o(1)\},
\end{aligned}
$$

where σ_v^{sk} denote the (s,k)-th element of $(\tilde{I}_{i1}\Sigma(t_{i1},t_{i1})\tilde{I}_{i1})^{-1}$ when $t_{i1} = t + h_v$,

$$
\xi_{skrl}(t) = 2f(t) \sum_{v=1}^{\min(s,k)} \sigma_v^{sk}(t) \int_{c_{1,v-1}}^{c_{1v}} z^{r+l} K_{h_s/h_q}^{\frac{1}{2}}(z) K_{h_k/h_q}^{\frac{1}{2}}(z)\, dz,
$$

and c_{1v}'s are defined in condition (C5) at the beginning of Section 3. Similarly, we can show that $\text{Var}^{1/2}(a_{r+1,l+1}^{(sk)}) = o(J_S h_q^{r+l})$. So, $a_{r+1,l+1}^{(sk)} = J_S h_q^{r+l}\xi_{skrl}(t)\{1+o_P(1)\}$, and consequently, we have

$$
A_m = J_S[(I_{q\times q}\otimes \widetilde{H}) S (I_{q\times q}\otimes \widetilde{H})]\{1+o_P(1)\}, \tag{S.2}
$$

where S is a $q(p+1)\times q(p+1)$ matrix with the $((s-1)(p+1)+r+1,(k-1)(p+1)+l+1)$-th element to be $\xi_{skrl}(t)$.

Similar to $(S.2)$, we can check that

$$B_m = J_S h_q^{-1}[(I_{q\times q}\otimes \widetilde{H})\bar{S}(I_{q\times q}\otimes \widetilde{H})]\{1+o_P(1)\} \tag{S.3}$$

where $\bar{S}$ is a $q(p+1)\times q(p+1)$ matrix with the $((s-1)(p+1)+r+1,(k-1)(p+1)+l+1)$-th element to be

$$\bar{\xi}_{skrl}(t) =$$
$$2f(t)\sum_{l_1=1}^{q}\sum_{l_2=1}^{q}\sum_{v=1}^{\min(s,k,l_1,l_2)}\sigma_v^{sl_1}(t)\sigma_{l_1l_2}(t)\sigma_v^{l_2k}(t)\int_{c_{1,v-1}}^{c_{1v}} z^{r+l}K_{c_{1s}}^{\frac{1}{2}}(z)K_{c_{1k}}^{\frac{1}{2}}(z)K_{c_{1l_1}}^{\frac{1}{2}}(z)K_{c_{1l_2}}^{\frac{1}{2}}(z)\,dz.$$

By combining $(S.1)-(S.3)$ and expression (4) in the paper, we have

$$\mathrm{Cov}\{\widehat{\boldsymbol{\mu}}(t)|\mathcal{F}_m\} = \frac{1}{J_S h_q}\left[(I_{q\times q}\otimes \boldsymbol{e}_1')S^{-1}\bar{S}S^{-1}(I_{q\times q}\otimes \boldsymbol{e}_1)\right] + o_P\left(\frac{1}{J_S h_q}\right).$$

Similar to the asymptotic expansion of B_m in $(S.3)$, we can show that

$$\begin{aligned}
\mathrm{Bias}\{\widehat{\boldsymbol{\mu}}(t)|\mathcal{F}_m\} &= (I_{q\times q}\otimes \boldsymbol{e}_1')[E(\widehat{\beta}|\mathcal{F}_m)-\beta] \\
&= (I_{q\times q}\otimes \boldsymbol{e}_1')A_m^{-1}\sum_{i=1}^{m}(I_{q\times q}\otimes X_i')W_iE[\mathbf{Y}_i-(I_{q\times q}\otimes X_i)\beta] \\
&= \frac{1}{(p+1)!}(I_{q\times q}\otimes \boldsymbol{e}_1')A_m^{-1}\sum_{i=1}^{m}(I_{q\times q}\otimes X_i')W_i\Big(\mu_1^{(p+1)}(t)(t_{i1}-t)^{p+1},\ldots, \\
&\quad \mu_1^{(p+1)}(t)(t_{iJ_1}-t)^{p+1},\ldots,\mu_q^{(p+1)}(t)(t_{i1}-t)^{p+1},\ldots,\mu_q^{(p+1)}(t)(t_{iJ_q}-t)^{p+1}\Big)' \\
&\quad \times\{1+o(1)\} \\
&= \frac{1}{(p+1)!}(I_{q\times q}\otimes \boldsymbol{e}_1')\{J_S[(I_{q\times q}\otimes \widetilde{H})S(I_{q\times q}\otimes \widetilde{H})]\{1+o_P(1)\}\}^{-1} \\
&\quad \times[J_S(I_{q\times q}\otimes \widetilde{H})Dh_q^{p+1}]\{1+o_P(1)\} \\
&= \frac{1}{(p+1)!}h_q^{p+1}[(I_{q\times q}\otimes \boldsymbol{e}_1')S^{-1}D]+o_P(h_q^{p+1}),
\end{aligned}$$

where D is a $[q(p+1)]$-dimensional vector with the $[(s-1)(p+1)+r+1]$-th element to be $f(t)\sum_{l=1}^{q}\mu_l^{(p+1)}(t)\xi_{slr(p+1)}(t)$. After combining the two expressions above for $\mathrm{Cov}\{\widehat{\boldsymbol{\mu}}(t)|\mathcal{F}_m\}$ and $\mathrm{Bias}\{\widehat{\boldsymbol{\mu}}(t)|\mathcal{F}_m\}$, the result (15) is proved.

1.2 Proof of Theorem 3.2

From the proof of Theorem 3.1, we can conclude that the initial estimator $\widetilde{\boldsymbol{\mu}}(t)$ has the consistency result stated in that theorem. That is,

$$\sup_{t\in[0,1]}|\widetilde{\mu}_l(t)-\mu_l(t)| = O_P(h_{\max}^{p+1}+1/(J_S h_{\max})^{\frac{1}{2}}), \quad \text{for } l=1,2,\ldots,q. \tag{S.4}$$

For $l_1, l_2 = 1, 2, \ldots, q$, let

$$M_{l_1,l_2}(s,t) = \frac{1}{J_{SS}}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k=1,k\neq j}^{J_i} K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t).$$

Then, by some arguments based on the Taylor's expansion, we have

$$\begin{aligned}
E(M_{l_1,l_2}(s,t)) &= E(K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t)) \\
&= \int \frac{1}{g_{l_1}}K\left(\frac{t_{ij}-s}{g_{l_1}}\right)\frac{1}{g_{l_2}}K\left(\frac{t_{ik}-t}{g_{l_2}}\right) f(t_{ij})f(t_{ik})\, dt_{ij}dt_{ik} \\
&= \int K(u)K(z)f(s+g_{l_1}u)f(t+g_{l_2}z)\, dudz \\
&= \int K(u)K(z)\left(f(s)+f'(s)g_{l_1}u+f''(s)g_{l_1}^2u^2+o(u^2)\right) \\
&\quad \times\left(f(t)+f'(t)g_{l_2}z+f''(t)g_{l_2}^2z^2+o(z^2)\right)\, dudz \\
&= f(s)f(t)(1+O_P(g_{\max}^2)).
\end{aligned}$$

Similarly, we have

$$
\begin{aligned}
\mathrm{Var}(M_{l_1,l_2}(s,t)) &= \frac{1}{J_{SS}}\mathrm{Var}(K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t)) \\
&= \frac{1}{J_{SS}}\int \frac{1}{g_{l_1}^2}K^2\left(\frac{t_{ij}-s}{g_{l_1}}\right)\frac{1}{g_{l_2}^2}K^2\left(\frac{t_{ik}-t}{g_{l_2}}\right)f(t_{ij})f(t_{ik})dt_{ij}dt_{ik} - \\
&\quad \frac{1}{J_{SS}}\left(\int \frac{1}{g_{l_1}}K\left(\frac{t_{ij}-s}{g_{l_1}}\right)\frac{1}{g_{l_2}}K\left(\frac{t_{ik}-t}{g_{l_2}}\right)f(t_{ij})f(t_{ik})dt_{ij}dt_{ik}\right)^2 \\
&= \frac{1}{J_{SS}g_{l_1}g_{l_2}}\int K(u)K(z)f(s+g_{l_1}u)f(t+g_{l_2}z)dudz - \\
&\quad \frac{1}{J_{SS}}\left(\int K(u)K(z)f(s+g_{l_1}u)f(t+g_{l_2}z)dudz\right)^2 \\
&= f(s)f(t)O_P(1/J_{SS}g_{\max}^2).
\end{aligned}
$$

Therefore,

$$
M_{l_1,l_2}(s,t)) = f(s)f(t)(1+O_P(g_{\max}^2 + 1/(J_{SS}^{\frac{1}{2}}g_{\max})). \tag{S.5}
$$

Let $\{y_{l_1}(t_{ij}) - \mu_{l_1}(t_{ij})\}\{y_{l_2}(t_{ik}) - \mu_{l_2}(t_{ik})\} = \sigma_{l_1l_2}(t_{ij},t_{ik}) + \varepsilon_{l_1l_2}(t_{ij},t_{ik})$, where $\varepsilon_{l_1l_2}(t_{ij},t_{ik})$ satisfies the conditions that $E(\varepsilon_{l_1l_2}(t_{ij},t_{ik})|t_{ij},t_{ik}) = 0$ and $\mathrm{Var}(\varepsilon_{l_1l_2}(t_{ij},t_{ik})|t_{ij},t_{ik}) = \omega_{l_1l_2}(t_{ij},t_{ik})$. Then,

$$
\frac{1}{J_{SS}}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k\neq j}^{J_i}(\widetilde{\varepsilon}_{ijl_1}\widetilde{\varepsilon}_{ikl_2} - \sigma_{l_1l_2}(s,t))K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ij}-t) =: I_1+I_2+I_3+I_4+I_5,
$$

where

$$
\begin{aligned}
I_1 &= \frac{1}{J_{SS}}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k\neq j}^{J_i}(\sigma_{l_1l_2}(t_{ij},t_{ik}) - \sigma_{l_1l_2}(s,t))K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t), \\
I_2 &= \frac{1}{J_{SS}}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k\neq j}^{J_i}\varepsilon_{l_1l_2}(t_{ij},t_{ik})K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t), \\
I_3 &= \frac{1}{J_{SS}}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k\neq j}^{J_i}(y_{l_1}(t_{ij}) - \mu_{l_1}(t_{ij}))(\mu_{l_2}(t_{ik}) - \widetilde{\mu}_{l_2}(t_{ik}))K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t), \\
I_4 &= \frac{1}{J_{SS}}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k\neq j}^{J_i}(\mu_{l_1}(t_{ij}) - \widetilde{\mu}_{l_1}(t_{ij}))(y_{l_2}(t_{ik}) - \mu_{l_2}(t_{ik}))K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t), \\
I_5 &= \frac{1}{J_{SS}}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k\neq j}^{J_i}(\mu_{l_1}(t_{ij}) - \widetilde{\mu}_{l_1}(t_{ij}))(\mu_{l_2}(t_{ik}) - \widetilde{\mu}_{l_2}(t_{ik}))K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t).
\end{aligned}
$$

Similar to (S.5), the following results can be derived using some arguments based on the Taylor's expansion and on the result (S.4):

$$
\begin{aligned}
I_1 &= f(s)f(t)O_P(g_{\max}^2 + 1/(J_{SS}^{\frac{1}{2}} g_{\max})), \\
I_2 &= f(s)f(t)O_P(1/(J_S g_{\max})), \\
I_3 &= f(s)f(t)O_P(h_{\max}^{p+1} + 1/(J_S h_{\max})^{\frac{1}{2}})O_P(1/(J_{SS}^{\frac{1}{2}} g_{\max})), \\
I_4 &= f(s)f(t)O_P(h_{\max}^{p+1} + 1/(J_S h_{\max})^{\frac{1}{2}})O_P(1/(J_{SS}^{\frac{1}{2}} g_{\max})), \\
I_5 &= f(s)f(t)O_P((h_{\max}^{p+1} + 1/(J_S h_{\max})^{\frac{1}{2}})^2)O_P(g_{\max}^2 + 1/(J_{SS}^{\frac{1}{2}} g_{\max})).
\end{aligned}
$$

It is obvious that, when $s \neq t$,

$$
\frac{1}{J_T}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k=1}^{J_i} K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t) = \frac{J_{SS}}{J_T}M_{l_1,l_2}(s,t)) + o_P(M_{l_1,l_2}(s,t))),
$$

$$
\frac{1}{J_T}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k=1}^{J_i} \widetilde{\varepsilon}_{ijl_1}\widetilde{\varepsilon}_{ikl_2}K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ik}-t) = \frac{J_{SS}}{J_T}(I_1+I_2+I_3+I_4+I_5)+o_P(I_1+I_2+I_3+I_4+I_5),
$$

where $J_T = \sum_{i=1}^m J_i^2$. Thus, in such cases, we have

$$
\sup_{s,t\in[0,1],s\neq t} |\widetilde{\sigma}_{l_1l_2}(s,t) - \sigma_{l_1l_2}(s,t)| = O_P(g_{\max}^2 + 1/(J_{SS}^{\frac{1}{2}} g_{\max})). \qquad \text{(S.6)}
$$

Let

$$
N_{l_1,l_2}(s,t) = \frac{1}{J_S}\sum_{i=1}^{m}\sum_{j=1}^{J_i} K_{g_{l_1}}(t_{ij}-s)K_{g_{l_2}}(t_{ij}-t),
$$

$$
\frac{1}{J_S}\sum_{i=1}^{m}\sum_{j=1}^{J_i}(\widetilde{\varepsilon}_{ijl_1}\widetilde{\varepsilon}_{ijl_2} - \sigma_{l_1l_2}(t,t))K_{g_{l_1}}(t_{ij}-t)K_{g_{l_2}}(t_{ij}-t) =: I_1' + I_2' + I_3' + I_4' + I_5',
$$

where

$$
\begin{aligned}
I_1' &= \frac{1}{J_S}\sum_{i=1}^{m}\sum_{j=1}^{J_i}(\sigma_{l_1l_2}(t_{ij},t_{ij})-\sigma_{l_1l_2}(t,t))K_{g_{l_1}}(t_{ij}-t)K_{g_{l_2}}(t_{ij}-t),\\
I_2' &= \frac{1}{J_S}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\varepsilon_{l_1l_2}(t_{ij},t_{ij})K_{g_{l_1}}(t_{ij}-t)K_{g_{l_2}}(t_{ij}-t),\\
I_3' &= \frac{1}{J_S}\sum_{i=1}^{m}\sum_{j=1}^{J_i}(y_{l_1}(t_{ij})-\mu_{l_1}(t_{ij}))(\mu_{l_2}(t_{ij})-\widetilde{\mu}_{l_2}(t_{ij}))K_{g_{l_1}}(t_{ij}-t)K_{g_{l_2}}(t_{ij}-t),\\
I_4' &= \frac{1}{J_S}\sum_{i=1}^{m}\sum_{j=1}^{J_i}(\mu_{l_1}(t_{ij})-\widetilde{\mu}_{l_2}(t_{ij}))(y_{l_2}(t_{ij})-\mu_{l_2}(t_{ij}))K_{g_{l_1}}(t_{ij}-t)K_{g_{l_2}}(t_{ij}-t),\\
I_5' &= \frac{1}{J_S}\sum_{i=1}^{m}\sum_{j=1}^{J_i}(\mu_{l_1}(t_{ij})-\widetilde{\mu}_{l_1}(t_{ij}))(\mu_{l_2}(t_{ij})-\widetilde{\mu}_{l_2}(t_{ij}))K_{g_{l_1}}(t_{ij}-t)K_{g_{l_2}}(t_{ij}-t).
\end{aligned}
$$

In cases when $s=t$, besides the above results, we have

$$
\begin{aligned}
N_{l_1,l_2}(t,t) &= O_P(f(t)/g_{\max}),\\
I_1' &= \frac{g_{\max}}{2}f(t)\{\sigma''_{l_1l_2}(t,t)+2f'(t)\}\int z^2K_{c_{3s}}(z)K_{c_{3l}}(z)dz+o_P(g_{\max}),\\
I_2' &= \omega_{l_1l_2}(t,t)f(t)O_P(1/\sqrt{J_Sg^3_{\max}}),\\
I_3' &= f(t)\sigma_{l_1l_1}(t,t)O_P((h^{p+1}_{\max}+1/\sqrt{J_Sh_{\max}})/\sqrt{J_Sg^3_{\max}}),\\
I_4' &= f(t)\sigma_{l_2l_2}(t,t)O_P((h^{p+1}_{\max}+1/\sqrt{J_Sh_{\max}})/\sqrt{J_Sg^3_{\max}}),\\
I_5' &= f(t)O_P((h^{p+1}_{\max}+1/\sqrt{J_Sh_{\max}})^2/g_{\max}).
\end{aligned}
$$

It is obvious that, when $s=t$,

$$
\begin{aligned}
&\frac{1}{J_T}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k=1}^{J_i}K_{g_{l_1}}(t_{ij}-t)K_{g_{l_2}}(t_{ik}-t)\\
&= \frac{J_{SS}}{J_T}M_{l_1,l_2}(t,t))+\frac{J_S}{J_T}N_{l_1,l_2}(t,t))\\
&= f(t)O_P(1/g_{\max}),
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{J_T}\sum_{i=1}^{m}\sum_{j=1}^{J_i}\sum_{k=1}^{J_i}\widetilde{\varepsilon}_{ijl_1}\widetilde{\varepsilon}_{ikl_2}K_{g_{l_1}}(t_{ij}-t)K_{g_{l_2}}(t_{ik}-t) \\
= & \frac{J_{SS}}{J_T}(I_1+I_2+I_3+I_4+I_5+\frac{J_S}{J_T}(I'_1+I'_2+I'_3+I'_4+I'_5) \\
= & f(t)O_P\left(g_{\max}+1/\sqrt{J_S g_{\max}^3}\right).
\end{aligned}
$$

Then, we have

$$
\sup_{s,t\in[0,1],s=t}|\widetilde{\sigma}_{l_1l_2}(s,t)-\sigma_{l_1l_2}(s,t)| = O_P(g_{\max}^2+1/(J_S g_{\max})^{\frac{1}{2}}). \tag{S.7}
$$

It is obvious that (S.6) and (S.7) are just (16) and (17) in the paper. So, Theorem 3.2 is proved.

1.3 Proof of Theorem 3.3

Theorem 3.2 shows that each element of $\widetilde{\Sigma}(s,t)$ converges to its counterpart in $\Sigma(s,t)$ as J_S tends to infinity, which implies that the elements of $\widetilde{\Sigma}(s,t)$ are all bounded in probability. Thus, by similar arguments to those in the proof of Theorem 3.2, the consistency result (18) about $\widehat{\boldsymbol{\mu}}(t,\widetilde{\Sigma})$ can be proved.

1.4 Proof of Theorem 3.4

The consistency results (19) and (20) can be proved in the same way as that of Theorem 3.2, after using the result in Theorem 3.3.

2 Some Extra Numerical Results

Table S.1 presents the results of the actual ATS_0 and ATS_1 values of the chart (13) in the same setup as that in Figure 1.

Table S.1: Actual ATS_0 (in the first row) and ATS_1 values of the chart (13) for detecting step mean shifts of size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when observation vectors within a subject are independent and normally distributed, the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are assumed known, $d = 2, 5$ or 10, $\omega = 0.001$, $\lambda_L = 0.05$, 0.1 or 0.2, and the nominal ATS_0 is 100.

Shifts	$\lambda = 0.05$			$\lambda = 0.1$			$\lambda = 0.2$		
$\boldsymbol{\delta}$	$d = 2$	$d = 5$	$d = 10$	$d = 2$	$d = 5$	$d = 10$	$d = 2$	$d = 5$	$d = 10$
0	100.79	101.38	100.82	100.91	99.74	100.99	100.99	100.64	99.21
1	81.56	67.37	54.40	81.91	67.59	59.78	84.55	74.42	66.46
2	56.34	37.66	25.22	55.15	36.56	26.18	56.94	40.88	30.65
3	40.58	23.72	15.05	37.39	21.72	14.32	36.77	22.84	15.61
4	29.87	16.93	10.46	27.17	15.01	9.30	25.93	14.43	9.29
5	70.38	52.77	39.01	69.13	52.37	42.88	72.48	59.56	50.40
6	52.45	34.25	22.61	50.43	32.65	23.13	51.18	36.19	27.14
7	38.51	22.87	14.47	35.78	20.88	13.42	35.36	21.64	14.76
8	43.59	26.73	17.39	41.11	24.85	16.81	40.31	26.31	18.33
9	27.46	15.60	9.71	24.58	13.48	8.46	22.60	12.53	8.08
10	29.47	17.11	10.69	26.83	14.99	9.44	24.76	14.29	9.14
11	21.55	12.23	7.54	18.93	10.25	6.33	16.71	9.02	5.63
12	63.89	43.85	31.65	61.81	43.55	33.53	63.16	49.86	39.99
13	27.13	15.35	9.50	24.04	13.23	8.27	22.23	12.52	7.86
14	36.72	22.28	13.99	33.83	19.78	12.80	32.38	20.37	13.76
15	25.42	14.61	8.98	22.84	12.35	7.68	20.29	11.42	7.26
16	17.80	10.03	6.21	15.38	8.05	5.05	12.81	6.78	4.26
17	29.10	17.03	10.63	26.29	14.63	9.23	24.08	13.83	9.08
18	22.55	12.82	8.00	19.86	10.70	6.72	17.53	9.58	6.05
19	29.69	17.20	10.79	26.28	14.99	9.41	24.44	14.22	9.16
20	17.02	9.55	5.91	14.44	7.63	4.79	12.14	6.34	3.96

Tables S.2, S.3 and S.4 present the actual ATS_0 and ATS_1 values of the chart (13) in the same setup as that in Figure 2 in the paper, except that $\lambda = 0.05$ in Table S.2 and $\lambda = 0.1$ in Table S.3.

Table S.2: Actual ATS_0 (in the first row) and ATS_1 values of the chart (13), along with their standard errors (in parentheses), for detecting step mean shifts of the size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are estimated from an IC data with m subjects, $m = 30, 50$ or 70, $d = 2, 5$ or 10, $\omega = 0.001$, $\lambda = 0.05$, and the nominal ATS_0 is 100.

Shifts		$d = 2$		Shifts		$d = 2$	
$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$	$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$
0	89.87(0.48)	92.78(0.46)	97.13(0.38)	11	21.57(0.09)	21.52(0.08)	21.52(0.07)
1	79.22(0.48)	79.68(0.43)	81.60(0.37)	12	61.84(0.30)	62.05(0.31)	63.22(0.27)
2	54.25(0.31)	55.77(0.28)	56.54(0.26)	13	26.16(0.16)	26.06(0.12)	26.91(0.10)
3	39.43(0.39)	40.29(0.37)	40.53(0.32)	14	35.85(0.19)	35.97(0.17)	36.24(0.15)
4	29.67(0.15)	29.76(0.13)	29.52(0.12)	15	22.90(0.10)	24.02(0.08)	25.06(0.07)
5	68.56(0.37)	68.83(0.36)	69.96(0.29)	16	16.60(0.09)	16.92(0.06)	17.56(0.06)
6	50.70(0.29)	51.10(0.27)	51.95(0.23)	17	28.00(0.08)	28.30(0.08)	29.87(0.07)
7	38.28(0.36)	38.91(0.31)	39.22(0.29)	18	21.85(0.06)	22.08(0.06)	22.53(0.05)
8	43.02(0.24)	43.27(0.22)	43.38(0.18)	19	27.12(0.11)	28.38(0.10)	29.29(0.08)
9	26.83(0.24)	27.30(0.21)	27.32(0.20)	20	17.11(0.05)	17.05(0.04)	17.01(0.04)
10	28.95(0.11)	30.00(0.10)	29.44(0.09)				
Shifts		$d = 5$		Shifts		$d = 5$	
$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$	$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$
0	90.84(0.46)	94.18(0.40)	99.46(0.36)	11	12.12(0.04)	12.18(0.03)	12.38(0.03)
1	64.40(0.40)	65.40(0.33)	66.45(0.31)	12	43.44(0.23)	43.48(0.19)	43.64(0.18)
2	35.30(0.22)	35.15(0.18)	37.65(0.16)	13	14.94(0.07)	15.10(0.06)	15.48(0.05)
3	23.82(0.25)	23.46(0.18)	23.97(0.16)	14	21.52(0.10)	21.89(0.08)	22.18(0.06)
4	16.96(0.08)	16.99(0.06)	16.97(0.05)	15	12.90(0.04)	13.87(0.04)	14.56(0.03)
5	50.18(0.26)	51.08(0.25)	52.71(0.22)	16	9.21(0.04)	9.72(0.03)	10.05(0.03)
6	32.60(0.20)	33.59(0.17)	34.78(0.15)	17	16.44(0.04)	16.79(0.04)	17.09(0.03)
7	23.09(0.22)	22.86(0.17)	22.71(0.13)	18	12.50(0.03)	12.72(0.03)	12.96(0.02)
8	27.04(0.13)	27.24(0.10)	26.75(0.10)	19	15.68(0.05)	16.53(0.04)	17.38(0.04)
9	15.38(0.12)	15.29(0.10)	15.60(0.08)	20	9.71(0.02)	9.73(0.02)	9.50(0.02)
10	17.47(0.05)	17.59(0.04)	17.23(0.04)				
Shifts		$d = 10$		Shifts		$d = 10$	
$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$	$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$
0	89.27(0.45)	93.37(0.37)	99.92(0.37)	11	7.55(0.02)	7.53(0.02)	7.54(0.02)
1	53.18(0.34)	53.84(0.32)	54.45(0.25)	12	31.25(0.16)	31.10(0.13)	31.88(0.13)
2	23.63(0.13)	24.33(0.12)	25.36(0.13)	13	9.31(0.04)	9.41(0.03)	9.47(0.03)
3	14.84(0.14)	15.14(0.12)	15.03(0.11)	14	13.75(0.05)	13.88(0.04)	13.95(0.04)
4	10.61(0.04)	10.53(0.03)	10.28(0.03)	15	7.97(0.02)	8.22(0.02)	8.62(0.02)
5	37.08(0.22)	38.71(0.16)	39.15(0.22)	16	5.74(0.02)	5.99(0.01)	6.27(0.01)
6	21.80(0.12)	22.54(0.10)	22.86(0.10)	17	10.36(0.02)	10.35(0.02)	10.71(0.02)
7	14.46(0.13)	14.73(0.10)	14.30(0.10)	18	7.76(0.01)	7.96(0.01)	8.03(0.01)
8	17.97(0.08)	17.87(0.07)	17.25(0.06)	19	9.76(0.02)	10.35(0.02)	10.75(0.02)
9	9.47(0.06)	9.63(0.05)	9.84(0.05)	20	6.04(0.01)	6.07(0.01)	5.97(0.01)
10	11.07(0.03)	10.99(0.02)	10.72(0.02)				

Table S.3: Actual ATS_0 (in the first row) and ATS_1 values of the chart (13), along with their standard errors (in parentheses), for detecting step mean shifts of the size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are estimated from an IC data with m subjects, $m = 30, 50$ or 70, $d = 2, 5$ or 10, $\omega = 0.001$, $\lambda = 0.1$, and the nominal ATS_0 is 100.

Shifts		$d=2$		Shifts		$d=2$	
$\boldsymbol{\delta}$	$m=30$	$m=50$	$m=70$	$\boldsymbol{\delta}$	$m=30$	$m=50$	$m=70$
0	90.02(0.54)	95.71(0.52)	98.98(0.49)	11	18.94(0.09)	18.83(0.08)	18.91(.07)
1	77.90(0.55)	78.90(0.49)	79.74(0.43)	12	59.75(0.33)	60.76(0.32)	61.45(0.30)
2	52.24(0.34)	53.67(0.33)	54.41(0.28)	13	23.36(0.15)	23.70(0.12)	24.15(0.11)
3	37.15(0.42)	37.25(0.37)	37.33(0.33)	14	32.88(0.21)	33.26(0.18)	33.84(0.16)
4	27.16(0.16)	27.04(0.13)	27.13(0.11)	15	20.21(0.10)	21.33(0.09)	22.75(0.07)
5	66.72(0.40)	67.76(0.39)	68.76(0.34)	16	14.06(0.08)	14.64(0.07)	15.33(.06)
6	48.45(0.33)	49.65(0.29)	50.72(0.26)	17	25.08(0.09)	25.79(0.09)	26.35(0.07)
7	34.67(0.38)	35.31(0.32)	35.59(0.29)	18	19.08(0.06)	19.44(0.06)	19.84(0.05)
8	40.17(0.25)	40.74(0.23)	41.18(0.19)	19	24.19(0.11)	25.42(0.10)	26.45(0.07)
9	24.11(0.25)	24.50(0.20)	24.57(0.20)	20	14.53(0.05)	14.69(0.04)	14.42(0.03)
10	27.14(0.11)	27.06(0.10)	26.84(0.08)				
Shifts		$d=5$		Shifts		$d=5$	
$\boldsymbol{\delta}$	$m=30$	$m=50$	$m=70$	$\boldsymbol{\delta}$	$m=30$	$m=50$	$m=70$
0	90.79(0.52)	95.20(0.45)	99.24(0.43)	11	10.12(0.04)	10.12(0.03)	10.25(0.03)
1	64.39(0.42)	65.20(0.38)	67.76(0.32)	12	42.09(0.23)	42.97(0.23)	43.52(0.20)
2	33.58(0.24)	34.85(0.20)	36.50(0.18)	13	12.73(0.07)	12.94(0.06)	13.28(0.05)
3	21.85(0.26)	21.58(0.20)	21.68(0.17)	14	19.22(0.10)	19.17(0.09)	19.53(0.07)
4	15.01(0.08)	14.97(0.06)	15.09(0.05)	15	10.82(0.04)	11.77(0.03)	12.57(0.03)
5	49.07(0.34)	51.24(0.30)	52.10(0.27)	16	7.40(0.03)	7.64(0.03)	7.97(0.03)
6	30.82(0.21)	31.70(0.18)	32.74(0.16)	17	14.09(0.04)	14.43(0.04)	14.70(0.03)
7	21.20(0.22)	20.92(0.18)	20.87(0.15)	18	10.08(0.03)	10.38(0.03)	10.59(0.02)
8	25.15(0.15)	25.36(0.11)	24.69(0.11)	19	13.37(0.05)	14.25(0.04)	14.89(0.04)
9	13.27(0.12)	13.16(0.09)	13.49(0.08)	20	7.76(0.02)	7.77(0.02)	7.67(0.01)
10	15.30(0.05)	15.40(0.05)	14.95(0.04)				
Shifts		$d=10$		Shifts		$d=10$	
$\boldsymbol{\delta}$	$m=30$	$m=50$	$m=70$	$\boldsymbol{\delta}$	$m=30$	$m=50$	$m=70$
0	90.08(0.54)	95.42(0.42)	99.60(0.37)	11	6.33(0.02)	6.32(0.02)	6.30(0.02)
1	58.07(0.4)	58.31(0.37)	59.29(0.29)	12	32.43(0.20)	32.37(0.17)	32.63(0.17)
2	23.35(0.16)	24.1(0.13)	26.25(0.15)	13	8.03(0.04)	7.94(0.03)	7.92(0.03)
3	13.96(0.15)	14.30(0.13)	14.25(0.12)	14	12.55(0.06)	12.67(0.05)	12.66(0.05)
4	9.43(0.04)	9.38(0.03)	9.33(0.04)	15	7.68(0.02)	7.66(0.02)	7.67(0.02)
5	38.64(0.24)	40.52(0.20)	42.92(0.23)	16	4.63(0.01)	4.59(0.01)	4.58(0.01)
6	21.41(0.15)	22.09(0.11)	22.91(0.13)	17	8.94(0.02)	9.17(0.02)	9.33(0.02)
7	13.49(0.14)	13.46(0.12)	13.45(0.11)	18	6.46(0.01)	6.48(0.01)	6.47(0.01)
8	17.42(0.10)	17.30(0.08)	17.15(0.07)	19	9.41(0.02)	9.39(0.02)	9.38(0.02)
9	8.23(0.06)	8.37(0.05)	8.34(0.05)	20	4.85(0.01)	4.87(0.01)	4.87(0.01)
10	9.83(0.03)	9.62(0.02)	9.42(0.02)				

Table S.4: Actual ATS_0 (in the first row) and ATS_1 values of the chart (13), along with their standard errors (in parentheses), for detecting step mean shifts of the size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are estimated from an IC dataset with m subjects, $m = 30, 50$ or 70, $d = 2, 5$ or 10, $\omega = 0.001$, $\lambda_L = 0.2$, and the nominal ATS_0 value is 100.

Shifts		$d = 2$		Shifts		$d = 2$	
$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$	$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$
0	91.37(0.69)	95.41(0.65)	98.79(0.53)	11	16.51(0.10)	16.67(0.08)	16.77(0.07)
1	79.29(0.65)	81.97(0.55)	83.74(0.51)	12	60.22(0.40)	61.80(0.39)	63.05(0.37)
2	51.60(0.42)	53.92(0.41)	56.37(0.34)	13	21.09(0.16)	21.09(0.13)	21.97(0.11)
3	35.99(0.45)	36.75(0.41)	36.93(0.36)	14	31.23(0.23)	31.58(0.22)	31.90(0.18)
4	25.28(0.17)	25.58(0.14)	25.86(0.12)	15	18.78(0.11)	19.39(0.10)	20.18(0.07)
5	67.69(0.48)	69.92(0.49)	72.12(0.41)	16	11.70(0.08)	12.21(0.07)	12.60(0.06)
6	47.80(0.35)	49.27(0.35)	50.50(0.30)	17	22.63(0.09)	23.04(0.10)	23.97(0.07)
7	34.41(0.40)	35.11(0.35)	35.45(0.33)	18	16.61(0.07)	16.91(0.07)	17.41(0.05)
8	39.74(0.29)	39.98(0.27)	40.11(0.23)	19	21.89(0.11)	23.07(0.10)	24.09(0.09)
9	21.99(0.27)	22.33(0.22)	22.51(0.22)	20	12.04(0.05)	12.17(0.04)	12.12(0.04)
10	24.03(0.13)	24.40(0.11)	24.67(0.10)				
Shifts		$d = 5$		Shifts		$d = 5$	
$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$	$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$
0	90.33(0.59)	94.89(0.58)	98.82(0.48)	11	8.93(0.04)	8.91(0.04)	8.98(0.04)
1	71.39(0.51)	71.85(0.48)	73.89(0.38)	12	46.64(0.32)	47.30(0.30)	48.62(0.23)
2	35.92(0.27)	37.49(0.25)	39.22(0.22)	13	11.79(0.07)	11.91(0.07)	12.25(0.06)
3	22.75(0.30)	22.33(0.23)	22.99(0.20)	14	19.34(0.14)	19.24(0.12)	19.68(0.09)
4	14.49(0.09)	14.52(0.07)	14.66(0.06)	15	11.65(0.04)	11.59(0.04)	11.65(0.04)
5	54.45(0.38)	57.19(0.32)	59.15(0.30)	16	6.12(0.03)	6.38(0.03)	6.59(0.03)
6	32.80(0.26)	34.89(0.22)	36.31(0.20)	17	13.21(0.05)	13.23(0.05)	13.37(0.04)
7	21.84(0.27)	21.51(0.21)	22.15(0.17)	18	9.14(0.03)	9.16(0.03)	9.21(0.02)
8	26.73(0.18)	27.00(0.15)	26.45(0.15)	19	14.44(0.06)	14.39(0.04)	14.45(0.04)
9	12.43(0.14)	12.28(0.10)	12.55(0.09)	20	6.45(0.02)	6.43(0.02)	6.50(0.01)
10	14.78(0.06)	14.52(0.06)	14.21(0.05)				
Shifts		$d = 10$		Shifts		$d = 10$	
$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$	$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$
0	88.95(0.57)	93.04(0.54)	98.96(0.45)	11	5.64(0.02)	5.65(0.02)	5.63(0.02)
1	65.83(0.43)	66.03(0.42)	66.72(0.36)	12	37.90(0.23)	38.50(0.19)	39.10(0.22)
2	29.09(0.20)	28.66(0.16)	29.06(0.17)	13	7.53(0.05)	7.41(0.04)	7.47(0.04)
3	15.18(0.20)	15.53(0.17)	15.44(0.15)	14	13.19(0.08)	13.33(0.07)	13.32(0.06)
4	9.41(0.05)	9.30(0.04)	9.34(0.05)	15	7.01(0.02)	6.97(0.02)	6.98(0.02)
5	49.48(0.31)	48.99(0.25)	49.67(0.27)	16	4.33(0.01)	4.30(0.01)	4.29(0.01)
6	26.99(0.17)	26.44(0.16)	26.80(0.17)	17	8.60(0.03)	8.57(0.02)	8.58(0.02)
7	14.60(0.18)	15.07(0.15)	14.93(0.14)	18	5.74(0.02)	5.76(0.01)	5.77(0.01)
8	18.82(0.14)	18.54(0.10)	18.90(0.10)	19	8.89(0.03)	8.85(0.02)	8.89(0.03)
9	7.85(0.07)	7.98(0.06)	7.98(0.06)	20	4.06(0.01)	4.08(0.01)	4.08(0.01)
10	9.17(0.04)	9.21(0.03)	9.29(0.03)				

Tables S.5, S.6 and S.7 present the actual ATS_0 and ATS_1 values of the procedures MDySS-M, MDySS-C, and MDySS-L in the same setup as that in Figure 3 in the paper, except that $\lambda_M = \lambda_C = \lambda_L = 0.05$ in Table S.5 and $\lambda_M = \lambda_C = \lambda_L = 0.1$ in Table S.6.

Table S.8 and Table S.9 present the actual ATS_0 and ATS_1 values in cases when

$$\begin{aligned}\Sigma(t,t) &= \text{diag}\{1, \exp(t), \frac{1}{1+t}, 2, \log(t+5)\} \times \begin{pmatrix} 1.0000 & 0.8000 & 0.6400 & 0.5120 & 0.4096 \\ 0.8000 & 1.0000 & 0.7000 & 0.4900 & 0.3430 \\ 0.6400 & 0.7000 & 1.0000 & 0.6000 & 0.3600 \\ 0.5120 & 0.4900 & 0.6000 & 1.0000 & 0.5000 \\ 0.4096 & 0.3430 & 0.3600 & 0.5000 & 1.0000 \end{pmatrix} \\ &\times \text{diag}\{1, \exp(t), \frac{1}{1+t}, 2, \log(t+5)\}, \quad t \in [0,1].\end{aligned}$$

Other settings in Table S.8 and Table S.9 are the same setup as that in plot (a) of Figure 2 and Figure 3 in the paper, respectively.

Table S.10 presents the actual ATS_0 and ATS_1 values of the chart (13) in the same setup as that in Figure 4 in the paper.

Table S.5: Actual ATS_0 (in the first row) and ATS_1 values of the procedures MDySS-M, MDySS-C, and MDySS-L, along with their standard errors (in parentheses), for detecting step mean shifts of the size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are estimated from an IC dataset with m subjects, $m = 70$, $d = 2, 5$ or 10, $\omega = 0.001$, $\lambda_M = \lambda_C = \lambda_L = 0.05$, and the nominal ATS_0 is 100.

Shifts		$d = 2$		Shifts		$d = 2$	
$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L	$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	97.87(0.36)	97.53(0.40)	97.13(0.38)				
1	87.95(0.32)	80.48(0.36)	81.60(0.37)	11	21.50(0.05)	21.51(0.06)	21.52(0.07)
2	67.24(0.26)	55.47(0.26)	56.54(0.26)	12	61.15(0.25)	62.99(0.26)	63.22(0.27)
3	41.98(0.24)	41.40(0.31)	40.53(0.32)	13	25.58(0.07)	26.11(0.10)	26.91(0.10)
4	31.09(0.10)	30.86(0.11)	29.52(0.12)	14	35.46(0.09)	35.52(0.15)	36.24(0.15)
5	74.37(0.27)	69.22(0.29)	69.96(0.29)	15	24.49(0.06)	24.22(0.07)	25.06(0.07)
6	56.20(0.21)	51.55(0.23)	51.95(0.23)	16	16.83(0.03)	16.95(0.06)	17.56(0.06)
7	38.89(0.19)	39.81(0.26)	39.22(0.29)	17	26.79(0.05)	27.28(0.07)	29.87(0.07)
8	45.96(0.17)	43.18(0.19)	43.38(0.18)	18	20.67(0.04)	21.52(0.05)	22.53(0.05)
9	26.02(0.10)	27.70(0.20)	27.32(0.20)	19	25.97(0.06)	26.90(0.07)	29.29(0.08)
10	28.74(0.08)	29.88(0.09)	29.44(0.09)	20	15.02(0.03)	16.33(0.03)	17.01(0.04)
Shifts		$d = 5$		Shifts		$d = 5$	
$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L	$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	99.52(0.33)	99.30(0.36)	99.46(0.36)				
1	76.10(0.28)	65.30(0.28)	66.45(0.31)	11	12.38(0.02)	12.11(0.03)	12.38(0.03)
2	47.65(0.20)	36.37(0.17)	37.65(0.16)	12	42.04(0.15)	43.07(0.18)	43.64(0.18)
3	25.07(0.13)	24.91(0.15)	23.97(0.16)	13	14.68(0.04)	14.97(0.06)	15.48(0.05)
4	17.97(0.05)	18.10(0.06)	16.97(0.05)	14	19.98(0.05)	20.95(0.07)	22.18(0.06)
5	55.71(0.21)	52.05(0.22)	52.71(0.22)	15	13.96(0.03)	13.92(0.03)	14.56(0.03)
6	36.78(0.14)	34.21(0.14)	34.78(0.15)	16	9.55(0.02)	9.62(0.03)	10.05(0.03)
7	22.93(0.10)	23.90(0.13)	22.71(0.13)	17	15.43(0.02)	16.59(0.03)	17.09(0.03)
8	29.05(0.09)	26.75(0.10)	26.75(0.10)	18	11.96(0.02)	12.09(0.02)	12.96(0.02)
9	14.97(0.05)	15.75(0.07)	15.60(0.08)	19	14.98(0.03)	15.26(0.04)	17.38(0.04)
10	16.93(0.04)	17.43(0.04)	17.23(0.04)	20	8.33(0.01)	9.05(0.01)	9.50(0.02)
Shifts		$d = 10$		Shifts		$d = 10$	
$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L	$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	99.21(0.35)	98.13(0.34)	99.92(0.37)				
1	65.45(0.28)	52.24(0.23)	54.45(0.25)	11	7.86(0.01)	7.42(0.02)	7.54(0.02)
2	33.66(0.13)	24.20(0.13)	25.36(0.13)	12	29.88(0.10)	30.09(0.14)	31.88(0.13)
3	16.03(0.08)	15.80(0.11)	15.03(0.11)	13	9.27(0.02)	9.20(0.03)	9.47(0.03)
4	11.28(0.02)	11.24(0.03)	10.28(0.03)	14	12.91(0.03)	13.11(0.04)	13.95(0.04)
5	41.65(0.18)	39.09(0.18)	39.15(0.22)	15	7.92(0.01)	7.91(0.02)	8.62(0.02)
6	24.86(0.09)	21.96(0.11)	22.86(0.10)	16	5.79(0.01)	5.83(0.01)	6.27(0.01)
7	14.30(0.06)	15.13(0.10)	14.30(0.10)	17	9.31(0.01)	9.60(0.02)	10.71(0.02)
8	19.25(0.05)	17.31(0.06)	17.25(0.06)	18	6.93(0.01)	7.40(0.01)	8.03(0.01)
9	9.44(0.03)	9.86(0.05)	9.84(0.05)	19	8.55(0.01)	9.37(0.02)	10.75(0.02)
10	10.62(0.02)	10.86(0.02)	10.72(0.02)	20	5.17(0.01)	5.58(0.01)	5.97(0.01)

Table S.6: Actual ATS_0 (in the first row) and ATS_1 values of the procedures MDySS-M, MDySS-C, and MDySS-L, along with their standard errors (in parentheses), for detecting step mean shifts of the size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are estimated from an IC dataset with m subjects, $m = 70$, $d = 2, 5$ or 10, $\omega = 0.001$, $\lambda_M = \lambda_C = \lambda_L = 0.1$, and the nominal ATS_0 is 100.

Shifts		$d = 2$		Shifts		$d = 2$	
$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L	$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	98.82(0.44)	97.84(0.49)	98.98(0.49)				
1	87.71(0.37)	78.98(0.41)	79.74(0.43)	11	19.46(0.05)	18.72(0.07)	18.91(.07)
2	66.83(0.30)	52.99(0.25)	54.41(0.28)	12	58.93(0.25)	59.85(0.29)	61.45(0.30)
3	40.41(0.23)	38.50(0.33)	37.33(0.33)	13	23.79(0.08)	23.53(0.11)	24.15(0.11)
4	29.25(0.10)	27.84(0.12)	27.13(0.11)	14	32.12(0.11)	32.44(0.16)	33.84(0.16)
5	73.80(0.35)	68.08(0.35)	68.76(0.34)	15	20.59(0.07)	21.33(0.07)	22.75(0.07)
6	54.90(0.24)	49.77(0.25)	50.72(0.26)	16	14.83(0.04)	14.59(0.05)	15.33(.06)
7	36.01(0.18)	36.97(0.27)	35.59(0.29)	17	23.73(0.06)	24.32(0.06)	26.35(0.07)
8	44.36(0.19)	41.12(0.20)	41.18(0.19)	18	17.79(0.04)	18.80(0.05)	19.84(0.05)
9	24.08(0.11)	24.76(0.19)	24.57(0.20)	19	22.90(0.06)	23.97(0.07)	26.45(0.07)
10	26.67(0.07)	26.80(0.09)	26.84(0.08)	20	13.29(0.03)	13.83(0.03)	14.42(0.03)
Shifts		$d = 5$		Shifts		$d = 5$	
$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L	$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	99.84(0.41)	99.48(0.44)	99.24(0.43)				
1	79.76(0.34)	67.80(0.33)	67.76(0.32)	11	10.95(0.02)	10.17(0.03)	10.25(0.03)
2	49.30(0.24)	35.55(0.20)	36.50(0.18)	12	42.52(0.16)	42.97(0.20)	43.52(0.20)
3	24.48(0.13)	23.15(0.16)	21.68(0.17)	13	13.28(0.04)	12.99(0.06)	13.28(0.05)
4	16.89(0.05)	16.06(0.06)	15.09(0.05)	14	18.53(0.05)	19.01(0.06)	19.53(0.07)
5	58.10(0.25)	51.70(0.26)	52.10(0.27)	15	11.77(0.03)	11.99(0.03)	12.57(0.03)
6	37.14(0.17)	32.04(0.16)	32.74(0.16)	16	7.67(0.02)	7.13(0.03)	7.97(0.03)
7	21.47(0.10)	22.22(0.14)	20.87(0.15)	17	12.90(0.03)	13.57(0.03)	14.70(0.03)
8	28.69(0.10)	25.24(0.12)	24.69(0.11)	18	9.59(0.02)	10.13(0.02)	10.59(0.02)
9	13.49(0.05)	13.73(0.08)	13.49(0.08)	19	12.49(0.03)	13.24(0.03)	14.89(0.04)
10	15.05(0.03)	15.38(0.04)	14.95(0.04)	20	7.17(0.01)	7.30(0.01)	7.67(0.01)
Shifts		$d = 10$		Shifts		$d = 10$	
$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L	$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	99.32(0.42)	99.34(0.40)	99.60(0.37)				
1	72.09(0.34)	59.55(0.31)	59.29(0.29)	11	6.91(0.01)	6.27(0.02)	6.30(0.02)
2	36.96(0.18)	24.87(0.16)	26.25(0.15)	12	31.44(0.12)	32.18(0.17)	32.63(0.17)
3	15.90(0.09)	15.17(0.13)	14.25(0.12)	13	8.36(0.02)	8.02(0.04)	7.92(0.03)
4	10.67(0.03)	10.10(0.04)	9.33(0.04)	14	11.66(0.03)	12.17(0.04)	12.66(0.05)
5	45.51(0.22)	41.48(0.25)	42.92(0.23)	15	6.79(0.01)	6.95(0.02)	7.67(0.02)
6	25.91(0.11)	22.23(0.13)	22.91(0.13)	16	4.85(0.01)	4.41(0.01)	4.58(0.01)
7	13.85(0.07)	14.45(0.10)	13.45(0.11)	17	8.33(0.01)	8.40(0.02)	9.33(0.02)
8	19.60(0.06)	16.98(0.07)	17.15(0.07)	18	5.83(0.01)	6.23(0.01)	6.47(0.01)
9	8.56(0.03)	8.57(0.05)	8.34(0.05)	19	8.09(0.01)	8.17(0.02)	9.38(0.02)
10	9.49(0.02)	9.72(0.02)	9.42(0.02)	20	4.36(0.00)	4.52(0.01)	4.87(0.01)

Table S.7: Actual ATS_0 (in the first row) and ATS_1 values of the procedures MDySS-M, MDySS-C, and MDySS-L, along with their standard errors (in parentheses), for detecting step mean shifts of the size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are estimated from an IC dataset with m subjects, $m = 70$, $d = 2, 5$ or 10, $\omega = 0.001$, $\lambda_M = \lambda_C = \lambda_L = 0.2$, and the nominal ATS_0 is 100.

Shifts	$d=2$			Shifts	$d=2$		
$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L	$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	98.45(0.48)	97.78(0.62)	98.79(0.53)				
1	89.92(0.41)	81.53(0.54)	83.74(0.51)	11	17.93(0.06)	16.36(0.07)	16.77(0.07)
2	69.04(0.34)	53.47(0.33)	56.37(0.34)	12	60.71(0.28)	61.28(0.37)	63.05(0.37)
3	40.72(0.25)	37.78(0.38)	36.93(0.36)	13	22.18(0.08)	21.56(0.10)	21.97(0.11)
4	28.67(0.12)	26.02(0.13)	25.86(0.12)	14	30.04(0.12)	31.14(0.19)	31.90(0.18)
5	75.19(0.34)	70.89(0.44)	72.12(0.41)	15	17.90(0.07)	18.07(0.07)	20.18(0.07)
6	56.37(0.29)	49.99(0.30)	50.50(0.30)	16	12.03(0.04)	11.23(0.06)	12.60(0.06)
7	35.74(0.21)	35.97(0.31)	35.45(0.33)	17	21.97(0.06)	22.25(0.07)	23.97(0.07)
8	44.90(0.21)	39.70(0.24)	40.11(0.23)	18	15.98(0.04)	16.43(0.06)	17.41(0.05)
9	22.64(0.12)	22.74(0.22)	22.51(0.22)	19	21.11(0.06)	21.79(0.08)	24.09(0.09)
10	24.37(0.08)	24.96(0.11)	24.67(0.10)	20	11.25(0.03)	11.48(0.03)	12.12(0.04)
Shifts	$d=5$			Shifts	$d=5$		
$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L	$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	99.37(0.44)	99.35(0.57)	98.82(0.48)				
1	83.76(0.33)	73.48(0.46)	73.89(0.38)	11	10.11(0.03)	8.88(0.04)	8.98(0.04)
2	55.10(0.26)	38.28(0.24)	39.22(0.22)	12	47.13(0.21)	47.56(0.25)	48.62(0.23)
3	25.99(0.15)	24.05(0.19)	22.99(0.20)	13	12.78(0.04)	11.93(0.06)	12.25(0.06)
4	17.30(0.07)	15.45(0.07)	14.66(0.06)	14	18.29(0.06)	19.01(0.08)	19.68(0.09)
5	62.89(0.28)	57.87(0.34)	59.15(0.30)	15	9.90(0.03)	9.70(0.03)	11.65(0.04)
6	40.58(0.20)	34.39(0.18)	36.31(0.20)	16	6.59(0.02)	6.06(0.03)	6.59(0.03)
7	22.17(0.12)	22.74(0.17)	22.15(0.17)	17	13.21(0.05)	13.23(0.05)	13.37(0.04)
8	30.91(0.14)	26.62(0.16)	26.45(0.15)	18	8.56(0.02)	8.89(0.03)	9.21(0.02)
9	13.02(0.06)	12.75(0.08)	12.55(0.09)	19	12.53(0.03)	13.18(0.04)	14.45(0.04)
10	14.73(0.04)	14.69(0.05)	14.21(0.05)	20	5.79(0.01)	6.00(0.02)	6.50(0.01)
Shifts	$d=10$			Shifts	$d=10$		
$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L	$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	98.62(0.40)	98.04(0.42)	98.96(0.45)				
1	78.95(0.33)	66.30(0.35)	66.72(0.36)	11	6.54(0.01)	5.53(0.02)	5.63(0.02)
2	44.28(0.24)	28.12(0.21)	29.06(0.17)	12	36.49(0.15)	37.57(0.24)	39.10(0.22)
3	18.10(0.11)	16.42(0.14)	15.44(0.15)	13	8.27(0.02)	7.47(0.04)	7.47(0.04)
4	11.36(0.03)	10.06(0.04)	9.34(0.05)	14	11.94(0.04)	12.59(0.06)	13.32(0.06)
5	52.96(0.26)	48.02(0.27)	49.67(0.27)	15	6.27(0.02)	6.00(0.02)	6.98(0.02)
6	30.45(0.16)	25.06(0.16)	26.80(0.17)	16	4.22(0.01)	3.83(0.01)	4.29(0.01)
7	15.16(0.09)	15.50(0.13)	14.93(0.14)	17	7.76(0.01)	7.95(0.02)	8.58(0.02)
8	22.76(0.09)	19.06(0.10)	18.90(0.10)	18	5.19(0.01)	5.51(0.01)	5.77(0.01)
9	8.44(0.04)	8.14(0.06)	7.98(0.06)	19	7.12(0.01)	7.63(0.02)	8.89(0.03)
10	9.41(0.02)	9.54(0.03)	9.29(0.03)	20	3.72(0.01)	3.74(0.01)	4.08(0.01)

Table S.8: Actual ATS_0 (in the first row) and ATS_1 values of the chart (13), along with their standard errors (in parentheses), for detecting step mean shifts of the size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are estimated from an IC dataset with m subjects, $m = 30, 50$ or 70, $d = 2$, $\omega = 0.001$, $\lambda_L = 0.2$, and the nominal ATS_0 is 100.

$\boldsymbol{\delta}$	$m = 30$	$m = 50$	$m = 70$
0	96.71(0.54)	97.72(0.46)	98.40(0.42)
1	81.51(0.58)	81.88(0.43)	83.06(0.38)
2	52.67(0.41)	53.53(0.28)	53.79(0.28)
3	36.85(0.41)	36.81(0.27)	36.47(0.24)
4	25.39(0.15)	25.38(0.13)	25.55(0.11)
5	68.70(0.50)	69.37(0.37)	70.03(0.32)
6	48.87(0.37)	49.15(0.25)	49.82(0.24)
7	35.31(0.36)	35.13(0.27)	35.28(0.22)
8	40.55(0.29)	40.67(0.22)	41.20(0.20)
9	22.56(0.22)	22.50(0.16)	22.35(0.14)
10	25.27(0.13)	25.29(0.09)	25.39(0.09)
11	16.55(0.09)	16.47(0.08)	16.62(0.07)
12	61.48(0.40)	62.14(0.31)	62.64(0.27)
13	21.42(0.13)	21.38(0.11)	21.65(0.10)
14	31.82(0.21)	31.71(0.17)	31.99(0.14)
15	18.55(0.10)	18.51(0.08)	18.58(0.07)
16	11.95(0.07)	11.93(0.05)	12.09(0.05)
17	23.34(0.11)	23.40(0.09)	23.47(0.08)
18	16.94(0.08)	17.01(0.06)	17.13(0.06)
19	22.76(0.13)	22.81(0.08)	23.04(0.08)
20	12.12(0.06)	12.24(0.05)	12.28(0.04)

Table S.9: Actual ATS_0 (in the first row) and ATS_1 values of the procedures MDySS-M, MDySS-C, and MDySS-L, along with their standard errors (in parentheses), for detecting step mean shifts of the size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are estimated from an IC dataset with m subjects, $m=70$, $d=2$, $\omega=0.001$, $\lambda_M=\lambda_C=\lambda_L=0.2$, and the nominal ATS_0 is 100.

$\boldsymbol{\delta}$	MDySS-C	MDySS-M	MDySS-L
0	99.69(100.3)	100.3(0.46)	98.40(0.42)
1	88.89(85.32)	83.32(0.41)	83.06(0.38)
2	63.51(57.80)	54.80(0.25)	53.79(0.28)
3	41.15(39.60)	37.60(0.24)	36.47(0.24)
4	26.33(28.43)	26.43(0.12)	25.55(0.11)
5	72.06(73.45)	70.45(0.33)	70.03(0.32)
6	50.68(52.37)	50.37(0.25)	49.82(0.24)
7	38.78(36.51)	35.51(0.21)	35.28(0.22)
8	42.39(42.71)	40.71(0.21)	41.20(0.20)
9	23.22(22.91)	22.61(0.13)	22.35(0.14)
10	23.89(26.71)	25.21(0.09)	25.39(0.09)
11	16.69(16.82)	16.52(0.07)	16.62(0.07)
12	56.88(62.66)	62.66(0.29)	62.64(0.27)
13	20.93(22.78)	21.78(0.09)	21.65(0.10)
14	29.33(33.26)	31.26(0.15)	31.99(0.14)
15	17.96(19.26)	18.76(0.07)	18.58(0.07)
16	11.50(11.76)	11.76(0.05)	12.09(0.05)
17	19.45(22.70)	22.70(0.08)	23.47(0.08)
18	15.71(16.72)	16.72(0.07)	17.13(0.06)
19	19.64(22.66)	22.66(0.08)	23.04(0.08)
20	11.02(11.55)	11.55(0.04)	12.28(0.04)

Table S.10: Actual ATS_0 (in the first row) and ATS_1 values of the chart (13), along with their standard errors (in parentheses), for detecting step mean shifts of the size $\boldsymbol{\delta}$ occurring at the initial time point, in cases when the IC mean function $\boldsymbol{\mu}(t)$ and the IC covariance matrix function $\Sigma(s,t)$ are estimated from an IC dataset with m_1 subjects, $m_1 = 30, 50$ or 70, $d = 2$, $\omega = 0.001$, $\lambda_L = 0.2$, and the nominal ATS_0 is 100.

$\boldsymbol{\delta}$	$m_1 = 30$	$m_1 = 50$	$m_1 = 70$
0	125.79(1.34)	110.72(0.60)	98.23(0.58)
1	109.82(1.23)	91.86(0.55)	88.41(0.50)
2	74.92(1.06)	71.12(0.55)	62.30(0.41)
3	56.88(1.02)	50.83(0.42)	41.11(0.42)
4	36.76(0.35)	28.03(0.22)	30.80(0.18)
5	98.97(1.13)	85.31(0.55)	76.40(0.47)
6	70.56(0.95)	65.49(0.50)	58.83(0.37)
7	55.34(1.04)	48.36(0.40)	39.54(0.39)
8	60.32(0.60)	46.87(0.35)	49.91(0.28)
9	37.96(0.54)	31.52(0.30)	24.65(0.24)
10	35.42(0.25)	29.68(0.17)	29.92(0.14)
11	24.35(0.16)	18.84(0.12)	19.84(0.09)
12	90.80(0.94)	76.25(0.56)	71.29(0.40)
13	32.12(0.31)	24.94(0.20)	26.82(0.16)
14	46.66(0.48)	40.65(0.23)	37.95(0.24)
15	27.05(0.28)	26.59(0.13)	23.54(0.11)
16	18.12(0.15)	14.85(0.10)	15.30(0.08)
17	33.79(0.28)	30.24(0.16)	28.50(0.14)
18	22.94(0.19)	21.25(0.10)	20.36(0.09)
19	33.22(0.34)	31.25(0.16)	28.77(0.14)
20	15.82(0.12)	14.97(0.06)	13.85(0.06)