Recent Research in Dynamic Screening System for Sequential Process Monitoring

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Abstract

Dynamic Screening problems arise from a variety of applications where we need to sequentially monitor the performance of individuals to detect any malfunction as early as possible. These applications have stimulated much recent research in the literature and a new methodology called dynamic screening system (DySS) has been developed. By comparing the longitudinal performance of a given individual with that of well-functioning individuals and by sequentially monitoring their difference, DySS can detect their significant difference early so that the potential damage to the given individual can be avoided or reduced. This paper aims to introduce recent research on DySS in different cases, including cases with univariate or multivariate performance variables and cases with independent or correlated observations.

Key Words: Correlation; Dynamic screening; Longitudinal data; Process monitoring; Sequential monitoring; Statistical process control.

1 Introduction

Dynamic screening (DS) problems encompass a wide range of applications where some performance variables (e.g., variables measuring the quality of a product, or risk factors of a disease) need to be sequentially monitored for early detection of faults and/or diseases. The DS problems are important because early detection of faults and/or diseases can warrant timely interventions so that adverse consequences (e.g., airplane crashes, occurrence of stroke or other deadly diseases) can be prevented or detected at early stages.

To solve the DS problems, one simple method is to construct pointwise confidence intervals of the mean performance variables from observed data of some well-functioning individuals. Then, the longitudinal performance of a given individual can be detected as abnormal if its observations of the performance variables are beyond the confidence intervals. In the longitudinal data analysis (LDA) literature, there has been some discussion about construction of such confidence intervals (e.g., Ma et al. 2012, Yao et al. 2005, and Zhao and Wu 2008). However, this confidence interval approach cannot monitor the given individual in a sequential way, and thus the history data cannot be used efficiently by this approach. Another potential approach to handle the DS problems is related to statistical process control (SPC). By a SPC control chart, we can sequentially monitor the longitudinal performance of an individual (cf., Hawkins and Olwell 1998, Montgomery 2009, Qiu 2014). But, this approach usually does not compare an individual with other individuals regarding their longitudinal performance, and it usually assumes that the observation distribution is unchanged over time when the longitudinal performance of an individual is in-control (IC) or satisfactory, which is often invalid in the DS problems. As an example, the distribution of our blood pressure readings would change when we get older even in cases when we are healthy and do not have any serious cardiovascular diseases. Therefore, both the confidence interval approach and the traditional SPC charts cannot solve the DS problems effectively.

Motivated by the SHARe Framingham Heart study, Qiu and Xiang (2014) suggested a so-called dynamic screening system (DySS) for solving the DS problem in univariate cases. The DySS method combines the strengths of LDA and SPC approaches by comparing the longitudinal performance of a given individual with that of some well-functioning individuals and by sequentially monitoring their difference. This method is designed mainly for cases when observations at different time points are independent. In recent several years, several alternative DySS methods have been proposed for cases when the observations at different time points are correlated and when observations are multivariate. In the next two sections, we will introduce these different versions of the DySS method in details. Some remarks about certain future research problems on this topic will conclude the article in the last section.

2 DySS Methods When Observations are Independent

In this section, we introduce some recent DySS methods for cases when process observations collected at different time points are assumed independent. Univariate cases are discussed in Subsection 2.1, multivariate cases are discussed in Subsection 2.2, and an improved version is discussed in Subsection 2.3.

2.1 Univariate cases

Qiu and Xiang (2014) suggested the first DySS method for univariate cases. This method was mainly discussed in cases when process observations collected at different time points were assumed independent, although correlated data cases were also briefly discussed. The method consists of the following three steps:

(i) Estimate the regular longitudinal pattern of the performance variable y from an observed longitudinal dataset of a group of m well-functioning individuals. This dataset is called *IC dataset* hereafter.

(ii) For a new individual to monitoring, standardize his/her observations using the estimated regular longitudinal pattern obtained in step (i).

(iii) Monitor the standardized observations of the new individual, and give a signal as soon as all available data suggest a significant shift in his/her longitudinal pattern from the estimated regular pattern.

These three steps will be briefly described below.

Assume that the longitudinal observations of the m well-functioning individuals included in the IC dataset follow the model

$$y(t_{ij}) = \mu(t_{ij}) + \sigma(t_{ij})\epsilon(t_{ij}), \text{ for } j = 1, 2, \ldots, n_i,\ i = 1, 2, \ldots, m, \qquad (1)$$

where $t_{ij} \in [0, T]$ are observation times, $y(t_{ij})$ is the jth observation of the ith individual, $\mu(\cdot)$ and $\sigma^2(\cdot)$ are the mean and variance functions of the performance variable $y(\cdot)$, and $\epsilon(\cdot)$ is the standardized noise with mean 0 and variance 1. Based on the local pth-order polynomial kernel smoothing, Qiu and Xiang (2014) suggested a four-step procedure for estimating $\mu(\cdot)$ and $\sigma^2(\cdot)$ in model (1). Their estimators are denoted as $\widehat{\mu}(\cdot)$ and $\widehat{\sigma}^2(\cdot)$, respectively.

For a given individual to monitor, assume that his/her observations are obtained at times $t_j^* \in [0, T]$, for $j = 1, 2, \ldots$. When the performance of that individual is IC, his/her observations should follow model (1). So, we define the standardized observations of that individual as

$$\widehat{\epsilon}(t_j^*) = \frac{y(t_j^*) - \widehat{\mu}(t_j^*)}{\widehat{\sigma}(t_j^*)}, \text{ for } j \geq 1. \tag{2}$$

When the performance of the given individual is IC, the standardized observations $\{\widehat{\epsilon}(t_j^*), j \geq 1\}$ should be independent of each other with the same mean 0 and the same variance 1. If the longitudinal performance of that individual becomes out-of-control (OC), e.g., his/her mean response starts to deviate from the IC mean function $\mu(\cdot)$, then this will be reflected in the distribution of the standardized observations.

Assume that we are interested in detecting an upward mean shift in the original performance variable y for the given individual, then we can apply a conventional control chart for detecting upward mean shifts to the standardized observations. In Qiu and Xiang (2014), an upward cumulative sum (CUSUM) chart was selected. This chart has the charting statistic defined as

$$C_j^+ = \max(0, C_{j-1}^+ + \widehat{\epsilon}(t_j^*) - k), \text{ for } j \geq 1, \tag{3}$$

where $C_0^+ = 0$ and $k > 0$ is an allowance constant. Then, the chart gives a signal of an upward mean shift when

$$C_j^+ > h_C, \tag{4}$$

where $h_C > 0$ is a control limit. For detecting a downward or arbitrary shift, a downward or two-sided CUSUM chart can be used. For such CUSUM charts and other alternative control charts that can also be considered here, read Chapters 3–6 in the book Qiu (2014).

The performance of a control chart, such as the one defined by (3)-(4), is usually measured by the IC and OC average run lengths (ARLs). However, these measures are appropriate only in cases when the observation times are equally spaced, which is often invalid in the DS applications. To overcome that difficulty, Qiu and Xiang (2014) suggested using the average time to signal (ATS) measure, described as follows. Let ω be a basic time unit, which is the largest time unit that all observation times are its integer multiples. Then, we define

$$n_j^* = t_j^*/\omega, \text{ for } j = 1, 2, \ldots,$$

where $n_0^* = t_0^* = 0$. For an individual whose longitudinal performance is IC, assume that a signal is given at the sth observation time. Then, the expected value of n_s^*, (i.e., $\mathrm{E}(n_s^*)$) is called the IC ATS, denoted as ATS_0. Similarly, for an individual whose longitudinal performance starts to deviate from the regular longitudinal pattern at the time point τ, the value $\mathrm{E}(n_s^*|n_s^* \geq \tau) - \tau$ is called OC ATS, denoted as ATS_1. Then, for the control chart (3)-(4), the value of ATS_0 can be specified beforehand, and the chart performs better for detecting a shift of a given size if its ATS_1 value is smaller. For the chart (3)-(4), the value k is often pre-specified, a large k value is good for detecting large shifts, and a small k value is good for detecting small shifts. Commonly used k values include 0.1, 0.2, 0.5 and 1.0. Once k is pre-specified, the value of h_C can be chosen such that a given value of ATS_0 is reached.

2.2 Multivariate cases

Qiu and Xiang (2015) proposed a multivariate DySS method. In multivariate cases, we have multiple performance variables that are included in the q-dimensional vector $\boldsymbol{y}$. In such cases, the model corresponding to the univariate model (1) becomes

$$\boldsymbol{y}(t_{ij}) = \boldsymbol{\mu}(t_{ij}) + \Sigma^{1/2}(t_{ij}, t_{ij})\boldsymbol{\epsilon}(t_{ij}), \text{ for } j = 1, 2, \ldots, n_i, \ i = 1, 2, \ldots, m,$$

where $\boldsymbol{y}(t_{ij}) = (y_1(t_{ij}), y_2(t_{ij}), \ldots, y_q(t_{ij}))'$ is the q-dimensional observation at time t_{ij}, $\boldsymbol{\mu}(t_{ij}) = (\mu_1(t_{ij}), \mu_2(t_{ij}), \ldots, \mu_q(t_{ij}))'$ and $\Sigma(t_{ij}, t_{ij})$ are the mean and covariance matrix of $\boldsymbol{y}(t_{ij})$, and $\boldsymbol{\epsilon}(t_{ij}) = (\epsilon_1(t_{ij}), \epsilon_2(t_{ij}), \ldots, \epsilon_q(t_{ij}))'$ is the q-dimensional error term with mean $\mathbf{0}$ and variance $I_{q\times q}$. By the estimation procedure proposed in Xiang et al. (2013), we can obtain estimators of $\boldsymbol{\mu}(t)$ and $\Sigma(t,t)$ that are denoted as $\widehat{\boldsymbol{\mu}}(t)$ and $\widehat{\Sigma}(t,t)$, respectively.

For a new individual to monitor, assume that its observations are obtained at $t_j^* \in [0, T]$, for $j \geq 1$. Then, similar to (2), his/her standardized observations are defined as

$$\widehat{\boldsymbol{\epsilon}}(t_j^*) = \Sigma^{-1/2}(t_j^*, t_j^*)\left[\boldsymbol{y}(t_j^*) - \widehat{\boldsymbol{\mu}}(t_j^*)\right].$$

Any mean shifts in $\boldsymbol{y}(t_j^*)$ will be reflected in $\widehat{\boldsymbol{\epsilon}}(t_j^*)$ and we can use a multivariate control chart to monitor $\{\widehat{\boldsymbol{\epsilon}}(t_j^*)\}$ in order to detect distributional shifts in $\{\boldsymbol{y}(t_j^*)\}$.

Qiu and Xiang (2015) adopted the multivariate exponentially weighted moving average (MEWMA) chart for monitoring $\{\widehat{\boldsymbol{\epsilon}}(t_j^*)\}$. The charting statistic of this chart is

$$\boldsymbol{E}_j = \lambda\widehat{\boldsymbol{\epsilon}}(t_j^*) + (1-\lambda)\boldsymbol{E}_{j-1}, \text{ for } j \geq 1,$$

where $\boldsymbol{E}_0 = \mathbf{0}$ and $\lambda \in (0, 1]$ is a weighting parameter. It gives a signal when

$$\boldsymbol{E}_j' \Sigma_{\boldsymbol{E}_j}^{-1} \boldsymbol{E}_j > h_E,$$

where $h_E > 0$ is a control limit, and $\Sigma_{\boldsymbol{E}_j}$ is the covariance matrix of $\boldsymbol{E}_j$. Since $\mathrm{Var}(\boldsymbol{\epsilon}(t_j^*))$ is asymptotically $I_{q\times q}$ and observations at different time points are assumed independent, $\Sigma_{\boldsymbol{E}_j}$ is approximately $\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2j}]I_{q\times q}$, or $\frac{\lambda}{2-\lambda}$ when j is large. So, the above expression can be replaced by

$$\frac{2-\lambda}{\lambda[1-(1-\lambda)^{2j}]}\boldsymbol{E}_j'\boldsymbol{E}_j > h_E,$$

or $\frac{2-\lambda}{\lambda}\boldsymbol{E}_j'\boldsymbol{E}_j > h_E$ for large values of j. When the dimensionality q is large, Qiu and Xiang (2015) suggested using the multivariate control charts that was based on variable selection and discussed in several papers, including Capizzi and Masarotto (2011), Wang and Jiang (2009), and Zou and Qiu (2009).

2.3 An improved version

As mentioned earlier, the observation times are often unequally spaced in the DS problems. In the DySS methods described above, we have accommodated the unequally spaced observation times in the performance evaluation metrics ATS_0 and ATS_1. However, the construction of the control charts (cf., (3)-(4)) has not accommodated the unequally spaced observation times yet. To overcome this limitation, Qiu, Zi and Zou (2017) proposed a control chart that takes into account the unequally spaced observation times in its construction, which is introduced below.

To detect mean shifts in the standardized observations $\{\widehat{\epsilon}(t_j^*), j \geq 1\}$, let us consider the following hypothesis testing problem: for a given $j \geq 1$,

$$H_0 : \mu_{\widehat{\epsilon}(t_j^*)} = 0 \text{ versus } H_a : \mu_{\widehat{\epsilon}(t_j^*)} = g(t_j^*) \neq 0.$$

At the current time point j, let us consider the following local constant kernel estimation procedure:

$$\operatorname*{argmin}_{a\in R} \sum_{\ell=1}^{j} \left[\widehat{\epsilon}(t_\ell^*) - a\right]^2 (1-\lambda)^{t_j^*-t_\ell^*}, \tag{5}$$

where $\lambda \in (0, 1]$ is a weighting parameter. The solution to a is the local constant kernel estimator of $g(t_j^*)$, which has the expression

$$\widehat{g}_\lambda(t_j^*) = \frac{\sum_{\ell=1}^{j} w_\ell(t_j^*)\widehat{\epsilon}(t_\ell^*)}{\sum_{\ell=1}^{j} w_\ell(t_j^*)},$$

where $w_\ell(t_j^*) = (1-\lambda)^{t_j^* - t_\ell^*}$. In (5), we estimate $g(t_j^*)$ using all observations collected at or before the current time t_j^*, they receive different weights at different time points, and the weights exponentially decay when the related observation times move away from t_j^*. From the weight formula $w_\ell(t_j^*) = (1-\lambda)^{t_j^* - t_\ell^*}$, it can be seen that unequally spaced observation times have been taken into account.

By considering a weighted generalized likelihood ratio test (WGLR), if we define

$$Q_{H_a}(t_j^*; \lambda) = \sum_{\ell=1}^{j} \left[\widehat{\epsilon}(t_\ell^*) - \widehat{g}(t_j^*)\right]^2 w_\ell(t_j^*)$$
$$Q_{H_0}(t_j^*; \lambda) = \sum_{\ell=1}^{j} \left[\widehat{\epsilon}(t_\ell^*)\right]^2 w_\ell(t_j^*),$$

then the WGLR test statistic for testing hypotheses in (5) is

$$W_\lambda(t_j^*) = Q_{H_0}(t_j^*; \lambda) - Q_{H_a}(t_j^*; \lambda) = \sum_{\ell=1}^{j} \left[2\widehat{\epsilon}(t_\ell^*) - \widehat{g}(t_\ell^*)\right] \widehat{g}(t_\ell^*) w_\ell(t_j^*).$$

A signal could be triggered at t_j^* if $W_\lambda(t_j^*)$ is large. By noticing the fact that the sequence $\{(W_\lambda(t_j^*), \widehat{g}(t_j^*)), j = 1, 2, \ldots\}$ forms a two-dimensional Markov chain given the design points, the test statistic $W_\lambda(t_j^*)$ can be computed recursively in the following way:

$$W_\lambda(t_j^*) = w_{j-1}(t_j^*) W_\lambda(t_{j-1}^*) + \left[2\widehat{\epsilon}(t_j^*) - \widehat{g}(t_j^*)\right]\widehat{g}(t_j^*),$$
$$\widehat{g}(t_j^*) = \left[\alpha_{j-1}\widehat{g}(t_{j-1}^*) + \widehat{\epsilon}(t_j^*)\right]/\alpha_j,$$

where $\alpha_j = \sum_{\ell=1}^{j} w_\ell(t_j^*) = w_{j-1}(t_j^*)\alpha_{j-1} + 1$. Since the distribution of $W_\lambda(t_j^*)$ is changing over time, it usually requires a quite long time for it to reach a steady state. Thus, the following standardized statistic would be preferred here:

$$W_\lambda^*(t_j^*) = \left[W_\lambda(t_j^*) - E_\lambda(t_j^*)\right] / \sqrt{V_\lambda(t_j^*)},$$

where $E_\lambda(t_j^*)$ and $V_\lambda(t_j^*)$ are respectively the mean and variance of $W_\lambda(t_j^*)$. A recursive algorithm for calculating $E_\lambda(t_j^*)$ and $V_\lambda(t_j^*)$ can also be found in Qiu, Zi and Zou (2017). Then the chart gives a signal when

$$W_\lambda^*(t_j^*) > h_W,$$

where $h_W > 0$ is a control limit. Proper selection of the parameter λ and the computation of h_W were discussed in Qiu, Zi and Zou (2017).

3 DySS Methods When Observations are Correlated

The DySS methods described in the previous section are for cases when process observations are independent of each other. In cases when process observations are correlated, they can still be used if their control limits are chosen properly from an IC dataset using numerical approaches such as the bootstrap algorithms. However, they may not be as effective as we would expect because the data correlation is not taken into account in their construction. In this section, we introduce some recent DySS methods proposed specifically for cases when process observations are correlated.

In model (1), assume that the covariance function of the longitudinal response $y(t)$ is $V(s,t) = \mathrm{Cov}(y(s), y(t))$, for $s, t \in [0, T]$. By the four-step model estimation procedure in Qiu and Xiang (2014), we can obtain an estimator of $V(s,t)$ from an IC dataset, denoted as $\widehat{V}(s,t)$. For a new individual to monitor, we assume that his/her observations are obtained at times $\{t_j^*, j = 1, 2, \ldots\}$, as in Section 2. Instead of monitoring the original observations $\{y(t_j^*), j = 1, 2, \ldots\}$, Li and Qiu (2016) suggested monitoring their de-correlated values as follows. Let t_j^* be the current time point. The covariance matrix of $\boldsymbol{y}_j = (y(t_1^*), y(t_2^*), \ldots, y(t_j^*))'$ can then be estimated by

$$\widehat{\Sigma}_{j,j} = \begin{pmatrix} \widehat{V}(t_1^*, t_1^*) & \cdots & \widehat{V}(t_1^*, t_j^*) \\ \vdots & \ddots & \vdots \\ \widehat{V}(t_j^*, t_1^*) & \cdots & \widehat{V}(t_j^*, t_j^*) \end{pmatrix}.$$

By the Cholesky decomposition, we have

$$\Phi_j \widehat{\Sigma}_{j,j} \Phi_j' = D_j^2,$$

where Φ_j is a $j \times j$ lower triangular matrix with all diagonal elements being 1, and $D_j = \mathrm{diag}\{d_1, \ldots, d_j\}$ is a diagonal matrix with all diagonal elements positive. Let $\widehat{\boldsymbol{\varepsilon}}_j = (\widehat{\varepsilon}(t_1^*), \ldots, \widehat{\varepsilon}(t_j^*))'$ and $\widehat{\varepsilon}(t_\ell^*) = y(t_\ell^*) - \widehat{\mu}(t_j^*)$, for $\ell = 1, 2, \ldots, j$. Then, if we define $\boldsymbol{e}_j^* = D_j^{-1} \Phi_j \widehat{\boldsymbol{\varepsilon}}_j$, we have $\mathrm{Var}(\boldsymbol{e}_j^*) = I_{j \times j}$. The last element of $\boldsymbol{e}_j^*$ is denoted as $e^*(t_j^*)$. Then, values in the sequence $\{e^*(t_1^*), e^*(t_2^*), \ldots\}$ are uncorrelated with each other, and they have the common mean 0 and the common variance 1. Li and Qiu (2016) suggested monitoring this sequence using a CUSUM chart. For instance, to detect an upward mean shift in the original observations, we can use the CUSUM chart

$$C_j^+ = \max(0, C_{j-1}^+ + e^*(t_j^*) - k),$$

where $C_0^+ = 0$ and $k > 0$ is an allowance constant, and the chart gives a signal when

$$C_j^+ > h,$$

where $h > 0$ is a control limit. In Li and Qiu (2016), it has been shown that the data de-correlation described above can be achieved by a recursive computation, which can speed up the computation significantly.

The above data-decorrelation procedure has several limitations, including i) extensive computation when j is large because computation of a large inverse matrix is involved at each time point, ii) requirement of a relatively large data storage, and iii) attenuation of a possible process mean shift as a price to pay for obtaining un-correlated observations. To partially overcome these limitations, You and Qiu (2017) proposed a modified version of the data de-correlation procedure. The main idea of the modified version is that instead of de-correlating all history data, we only de-correlate a small portion of the history data observed after the previous time that the related CUSUM chart re-starts its charting statistic. Thus, the unnecessary de-correlation for the majority portion of the history data is avoided in this algorithm. To this end, You and Qiu used the concept of sprint length that was originally defined in Chatterjee and Qiu (2009) as follows:

$$T_j = \begin{cases} 0, & \text{if } C_j^+ = 0, \\ k, & \text{if } C_j^+ \neq 0, \ldots, C_{j-k+1}^+ \neq 0, C_{j-k}^+ = 0. \end{cases}$$

Then, we only need to de-correlate the current residual $\widehat{\varepsilon}(t_j^*)$ with residuals within the sprint length T_j of the current time point t_j^*. It has been shown that the computation of this modified version is much faster than that of the original method.

A multivariate extension of the data decorrelation method discussed in Li and Qiu (2016) was discussed in Li and Qiu (2017). Again, this multivariate data decorrelation method can be simplified using the sprint length idea described above, which has not been discussed yet in the literature.

4 Conclusions

We have introduced some recent methodologies for solving the DS problems in different cases, including the ones with univariate or multivariate performance variables and the ones with independent or correlated process observations. Because the DS problems have broad applications in industries, public health, medical studies, and many other areas, the introduced DySS methods should have a great potential to provide a major statistical tool for properly handling these applications.

The research topic on DySS is still new, and there are many open research problems. For instance, the classical performance measures for control charts, including ARL and ATS (see the related discussions in Section 2), accommodates the signal times well. But, they cannot reflect the overall false positive and false negative performance of the DySS methods. On the other hand, the regular false positive rate (FPR) and false negative rate (FNR) cannot accommodate the signal times well. So, a new performance metric is needed for the DySS methods. Also, there could be different covariates involved in the DS problems in practice. The existing DySS methods discussed in this paper have not accommodated such covariates properly yet.

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References

Capizzi, G., and Masarotto, G. (2011), "A least angle regression control chart for multidimensional data," *Technometrics*, **53**, 285–296.

Chatterjee, S., and Qiu, P. (2009), "Distribution-free cumulative sum control charts using bootstrap-based control limits," *Annals of Applied Statistics*, **3**, 349–369.

Hawkins, D.M., and Olwell, D.H. (1998), *Cumulative Sum Charts and Charting for Quality Improvement*, New York: Sprinter-Verlag.

Li, J., and Qiu, P. (2016), "Nonparametric dynamic screening system for monitoring correlated longitudinal data," *IIE Transactions*, **48**, 772–786.

Li, J., and Qiu, P. (2017), "Construction of an efficient multivariate dynamic screening system," *Quality and Reliability Engineering International*, in press.

Ma, S., Yang, L., and Carroll, R. (2012), "A simultaneous confidence band for sparse longitudinal regression," *Statistica Sinica*, **22**, 95–122.

Montgomery, D. C. (2009), *Introduction To Statistical Quality Control (6th edition)*, New York: John Wiley & Sons.

Qiu, P. (2014), *Introduction to Statistical Process Control,* Boca Raton, FL: Chapman Hall/CRC.

Qiu, P., and Xiang, D. (2014), "Univariate dynamic screening system: an approach for identifying individuals with irregular longitudinal behavior," *Technometrics*, **56**, 248-260.

Qiu, P., and Xiang, D. (2015), "Surveillance of cardiovascular diseases using a multivariate dynamic screening system," *Statistics in Medicine*, **34**, 2204–2221.

Qiu, P., Zi, X., and Zou, C. (2017), "Nonparametric dynamic curve monitoring," accepted with minor revisions by *Technometrics*.

Wang, K., and Jiang, W. (2009), "High-dimensional process monitoring and fault isolation via variable selection," *Journal of Quality Technology*, **41**, 247–258.

Xiang, D., Qiu, P., and Pu, X. (2013), "Nonparametric regression analysis of multivariate longitudinal data," *Statistica Sinica*, **23**, 769–789.

Yao, F., Müller, H.G., and Wang, J.L. (2005), "Functional data analysis for sparse longitudinal data," *Journal of the American Statistical Association*, **100**, 577–590.

You, L., and Qiu, P. (2017), "Fast computing for dynamic screening systems when analyzing correlated data," in review.

Zhao, Z., and Wu, W. (2008), "Confidence bands in nonparametric time series regression," *Annals of Statistics*, **36**, 1854–1878.

Zou, C., and Qiu, P. (2009), "Multivariate statistical process control using LASSO", *Journal of the American Statistical Association*, **104**, 1586–1596.