Some Recent Studies in Statistical Process Control

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Abstract Statistical process control (SPC) charts are widely used in manufacturing industries for quality control and management. They are used in more and more other applications, such as internet traffic monitoring, disease surveillance, and environmental protection. Traditional SPC charts designed for monitoring production lines in manufacturing industries are based on the assumptions that observed data are independent and identically distributed with a parametric in-control distribution. These assumptions, however, are rarely valid in practice. Therefore, recent SPC research focuses mainly on development of new control charts that are appropriate to use without these assumptions. In this article, we briefly introduce some recent studies on nonparametric SPC, control charts for monitoring dynamic processes, and spatio-temporal process monitoring. Control charts developed in these directions have found broad applications in practice.

1 Introduction

Since the first control chart suggested by Shewhart (1931), statistical process control (SPC) charts have become a basic and powerful tool for quality control and management in manufacturing industries. Many different control charts have been developed in the past more than eighty years. These charts are mainly in the following four categories: Shewhart charts, cumulative sum (CUSUM) charts, exponentially weighted moving average (EWMA) charts, and charts based on change-point detection (CPD). For systematic descriptions about the basics of these control charts, see books Hawkins and Olwell (1998), Montgomery (2012), and Qiu (2014).

Conventional control charts in the SPC literature are developed under the routine assumptions that process observations are independent and identically distributed (i.i.d.) with a parametric in-control (IC) distribution (e.g., normal). These assump-

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tions are rarely valid in practice. For instance, process observations collected at different time points could be serially correlated. Distributions of certain quality variables could be skewed and inappropriate to describe by a normal or another parametric distribution. In manufacturing industries, it might be reasonable to assume that the IC distribution of process observations does not change over time. But, in some other applications (e.g., monitoring of incidence rates of influenza over time), the process IC distribution usually changes over time and space, due to seasonality and other reasons. It has been well demonstrated in the literature that the conventional control charts designed under the above routine assumptions would not be reliable if one or more of their model assumptions are violated (e.g., Apley and Tsung 2002, Capizzi 2015, Chakraborti et al. 2001, Qiu 2018a, Qiu and Hawkins 2001, Qiu and Xiang 2014, Wardell et al. 1994). So, much recent research effort in the SPC community has been made in developing more flexible control charts. This chapter aims to describe some of them in the research directions of nonparametric SPC, control charts for monitoring dynamic processes, and spatio-temporal process monitoring.

SPC can be roughly divided into two phases. In Phase I, we try to adjust a process under monitoring to make it run stably and satisfactorily (or IC), which usually happens when the process is first monitored (e.g., a machine for production is first installed). To know whether the process is IC, a Phase I control chart needs to be applied to a small dataset collected from the process, and adjust the process if it is not IC. This control-and-adjustment process usually needs to be repeated several times until it is certain that the process is IC. Then, a Phase II control chart can be designed properly, based on an IC dataset collected after Phase I SPC, for online process monitoring. This chapter mainly introduce methods for Phase II SPC, although many methods introduced here can be modified easily for Phase I SPC. Also, in some SPC applications, the process under monitoring cannot be adjusted easily (e.g., when monitoring incidence rates of influenza or satellite images of earth surface in a given region). In such cases, Phase I SPC may not be relevant.

The remaining parts of the chapter are organized as follows. In Section 2, some conventional control charts are briefly introduced. Then, nonparametric SPC for cases when a parametric form is inappropriate or unavailable for describing the process distributions is discussed in Section 3. In Section 4, monitoring of processes with time-varying IC distributions (or dynamic processes) is discussed, followed by a discussion about spatio-temporal process monitoring in Section 5. Finally, some remarks conclude the chapter in Section 6.

2 Basic Control Charts

As mentioned in Section 1, early control charts in the SPC literature are in the framework of Shewhart charts that are described briefly in this section. Assume that X is a univariate quality variable in a specific process monitoring problem, it is continuous numerical, and its IC distribution is $N(\mu_0, \sigma^2)$. A batch of process

observations obtained at the nth time point is denoted as

$$X_{n1}, X_{n2}, \ldots, X_{nm},$$

where $m \geq 2$ is the batch size. To test whether the process is IC at the nth time point, it is natural to use the following z-test: the process is declared out-of-control (OC) if

$$\overline{X}_n > \mu_0 + Z_{1-\alpha/2}\frac{\sigma}{\sqrt{m}} \qquad \text{or} \qquad \overline{X}_n < \mu_0 - Z_{1-\alpha/2}\frac{\sigma}{\sqrt{m}},$$

where $\overline{X}_n$ is the sample mean of $\{X_{n1}, X_{n2}, \ldots, X_{nm}\}$, and $Z_{1-\alpha/2}$ is the $(1-\alpha/2)$th quantile of the $N(0,1)$ distribution. In practice, both μ_0 and σ could be unknown, and they need to be estimated from an IC dataset $\{(X^*_{i1}, X^*_{i2}, \ldots, X^*_{im}), i = 1,2,\ldots,M\}$. Let $\overline{X}^*_i$ and R^*_i be the sample mean and sample range of $(X^*_{i1}, X^*_{i2}, \ldots, X^*_{im})$, for $i = 1,2,\ldots,M$, and $\overline{\overline{X}}^*$ and $\overline{R}^*$ be the averages of $\{\overline{X}^*_i, i = 1,2,\ldots,M\}$ and $\{R^*_i, i = 1,2,\ldots,M\}$, respectively. Then, it can be checked that $\overline{\overline{X}}^*$ is an unbiased estimator of μ_0 and $\overline{R}^*/d_1(m)$ is an unbiased estimator of σ, where $d_1(m)$ is a constant that depends on m. When $m = 2,3,4,5$, $d_1(m) = 1.128, 1.693, 2.059$ and 2.326, respectively. See Table 3.1 in Qiu (2014) for more values of $d_1(m)$. After replacing μ_0 and σ by their estimates in the z-test, we obtain the Shewhart chart. So, the Shewhart chart declares a process mean shift at the nth time point if

$$\overline{X}_n > \overline{\overline{X}}^* + Z_{1-\alpha/2}\frac{\overline{R}^*}{d_1(m)\sqrt{m}} \qquad \text{or} \qquad \overline{X}_n < \overline{\overline{X}}^* - Z_{1-\alpha/2}\frac{\overline{R}^*}{d_1(m)\sqrt{m}}. \quad (1)$$

In manufacturing industries, we often choose $\alpha = 0.0027$. In such cases, $Z_{1-\alpha/2} = 3$. The popular terminology "six-sigma" in quality control and management is related directly to the above design of the Shewhart chart. Namely, the performance of a production process at time n can be considered IC if $\overline{X}_n$ is within an interval of six-sigma wide that is centered at μ_0, where sigma is the standard deviation of $\overline{X}_n$.

There are many different versions of the Shewhart chart (1) for detecting mean shifts. For instance, instead of using sample ranges for estimating the IC process standard deviation, we can also use sample standard deviations. The Shewhart chart (1) is constructed based on batch data with the batch size $m \geq 2$. When $m = 1$, there is only one observation at each time point. In such cases, the data are called individual observation data. There are some Shewhart charts suggested in the literature for monitoring individual observation data, some of which are based on the moving-window idea that the sample means and sample ranges used in constructing the Shewhart chart (1) are calculated from batches of observed data in different windows of observation times. There are many Shewhart charts in the literature suggested for detecting shifts in process variance, for monitoring binary or count data, and for many other purposes. See Chapter 3 in Qiu (2014) for a detailed description.

The Shewhart chart (1) makes a decision whether a process is IC at a given time point based on the observed data at that time point only. It is thus ineffective for Phase II process monitoring in most cases, because the observed data in the past can also provide helpful information about the process performance at the current

time point and such information is ignored completely by the Shewhart chart. To overcome this limitation, Page (1954) suggested the first CUSUM chart, and then many different CUSUM charts have been suggested in the literature for different purposes (cf., Hawkins and Olwell 1998, Qiu 2014). Next, we briefly describe the basic CUSUM chart for detecting a mean shift of a normal-distributed process. Assume that the IC process distribution is $N(\mu_0, \sigma^2)$, and the process observations for online monitoring are $\{X_n, n = 1, 2, \ldots\}$. Then, the CUSUM charting statistics for detecting a mean shift are defined by

$$\begin{aligned} C_n^+ &= \max(0, C_{n-1}^+ + (X_n - \mu_0)/\sigma - k), \\ C_n^- &= \min(0, C_{n-1}^- + (X_n - \mu_0)/\sigma + k), \qquad \text{for } n \geq 1, \end{aligned} \tag{2}$$

where $C_0^+ = C_0^- = 0$, and $k > 0$ is an allowance constant. The chart gives a signal when

$$C_n^+ > \rho_C \qquad \text{or} \qquad C_n^- < -\rho_C, \tag{3}$$

where $\rho_C > 0$ is a control limit. In the above CUSUM chart (2)-(3), the allowance constant k is usually pre-specified. Then, the control limit ρ_C is determined so that the IC average run length (ARL), denoted as ARL_0, equals a given value, where ARL_0 is defined as the average number of observation times from the beginning of process monitoring to a signal when the process is IC. From (2), it can be seen that (i) the charting statistics C_n^+ and C_n^- make use of the cumulative information in all available data by the current time point n, and (ii) they re-start from 0 each time when the cumulative information suggests no significant evidence of a mean shift in the sense that $C_{n-1}^+ + (X_n - \mu_0)/\sigma < k$ and $C_{n-1}^- + (X_n - \mu_0)/\sigma > -k$. The re-starting mechanism of the CUSUM chart makes it possess a good theoretical property that it has the smallest value of OC ARL, denoted as ARL_1, among all control charts that have the same ARL_0 value (cf., Moustakides 1986), where ARL_1 is defined as the average number of observation times from the occurrence of a real mean shift to a signal after the process becomes OC.

Although the CUSUM chart (2)-(3) has good properties for process monitoring, it is quite complicated to use, especially at the time when computing was expensive in the 1950s when the chart was first suggested. An alternative but simpler chart is the EWMA chart, first suggested by Roberts (1959) in the first volume of *Technometrics*. In the same setup as that for the CUSUM chart (2)-(3), the EWMA charting statistic is defined as

$$E_n = \lambda X_n + (1 - \lambda) E_{n-1}, \tag{4}$$

where $E_0 = \mu_0$, and $\lambda \in (0, 1]$ is a weighting parameter. From (4), it is easy to check that

$$E_n = \lambda \sum_{i=1}^{n} (1 - \lambda)^{n-i} X_i + (1 - \lambda)^n \mu_0, \tag{5}$$

and when the process is IC up to the current time point n, we have

$$E_n \sim N\left(\mu_0, \frac{\lambda}{2 - \lambda}\left[1 - (1 - \lambda)^{2n}\right]\sigma^2\right). \tag{6}$$

Equation (5) implies that E_n is a weighted average of μ_0 and all available observations up to n, and the weight received by X_i decays exponentially fast when i moves away from n. So, it is easy to study the IC properties of E_n, including the IC distribution given in (6). Based on Expression (6), the EWMA chart gives a signal of process mean shift when

$$|E_n - \mu_0| > \rho_E \sigma \sqrt{\frac{\lambda}{2-\lambda}\,[1-(1-\lambda)^{2n}]}, \tag{7}$$

where $\rho_E > 0$ is a control limit. In the EWMA chart (7), the weighting parameter λ is usually pre-specified, and the control limit ρ_E is chosen such that a given ARL_0 value is reached.

To use the Shewhart, CUSUM and EWMA charts described above, the IC parameters μ_0 and σ should be known or estimated in advance, which is inconvenient for certain applications. To overcome this limitation, Hawkins et al. (2003) suggested a CPD chart described below. For process observations $X_1, X_2, \ldots, X_n$, it is assumed that they follow the following change-point model:

$$X_i = \begin{cases} \mu_0 + \varepsilon_i, & \text{if } i = 1,2,\ldots,r, \\ \mu_1 + \varepsilon_i, & \text{if } i = r+1, r+2, \ldots, n, \end{cases}$$

where r is a change-point, and $\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}$ is a sequence of i.i.d. random variables with the common distribution $N(0, \sigma^2)$. Then, the likelihood ratio test statistic for testing the existence of a change-point is

$$T_{max,n} = \max_{1 \le j \le n-1} \sqrt{\frac{j(n-j)}{n}}\, \left| \overline{X}_j - \overline{X}'_j \right| \Big/ \widetilde{S}_j, \tag{8}$$

where $\overline{X}_j$ and $\overline{X}'_j$ are sample means of the first j and the remaining $n-j$ observations in $\{X_1, X_2, \ldots, X_n\}$, respectively, and $\widetilde{S}_j^2 = \sum_{i=1}^{j} (X_i - \overline{X}_j)^2 + \sum_{i=j+1}^{n} (X_i - \overline{X}'_j)^2$. The CPD chart gives a signal of mean shift when

$$T_{max,n} > \rho_n, \tag{9}$$

where $\rho_n > 0$ is a control limit that may depend on n. After a signal is given, an estimate of the change-point r is given by the maximizer found in (8). Hawkins et al. (2003) provided formulas for computing the values of ρ_n used in (9) for several commonly used ARL_0 values.

The description about the four types of basic control charts given above is for detecting process mean shifts when process observations are univariate. There are many different versions of each type for detecting shifts in process mean, variance and other aspects of the process distribution. There are many control charts for monitoring multivariate processes as well. See references, such as Crosier (1988), Gan (1993), Hawkins (1987, 1991), Hawkins et al. (2007), Healy (1987), Lowry et al. (1992), Mason et al. (2001), Sparks (2000), Woodall and Ncube (1985), Zamba and Hawkins (2006), Zou and Qiu (2009), and many more.

3 Nonparametric Control Charts

The basic control charts discussed in Section 2 are all based on the assumption that IC process observations follow a parametric (e.g., normal) distribution. In practice, this assumption is rarely valid and distributions of quality variables are often skewed and difficult to describe well by any parametric distributions. It has been well demonstrated in the literature that conventional control charts are unreliable to use in cases when their distributional assumptions are invalid (e.g., Capizzi 2015, Chakraborti et al. 2001, Chakraborti et al. 2015, Hackl and Ledolter 1992, Qiu 2008, Qiu 2018a, Qiu and Hawkins 2001, 2003). Thus, distribution-free or nonparametric SPC is under rapid development in the past twenty years. Some fundamental nonparametric SPC charts are described below in this section.

The first type of nonparametric SPC charts makes use of the ordering or ranking information in process observations collected at different time points. Let us first discuss univariate cases when there is only one quality variable X involved in process monitoring. Assume that the batch of observed data at the nth time point is $\{X_{n1}, X_{n2}, \ldots, X_{nm}\}$, for $n \geq 1$. Let ξ_0 be the median of the IC process distribution, and R_{nj} be the rank of $|X_{nj} - \xi_0|$ in the sequence $\{|X_{n1} - \xi_0|, |X_{n2} - \xi_0|, \ldots, |X_{nm} - \xi_0|\}$. Then, the sum of the Wilcoxon signed-ranks within the nth batch of observations is defined as

$$\psi_n = \sum_{j=1}^{m} \text{sign}(X_{nj} - \xi_0) R_{nj}, \tag{10}$$

where sign(u) = -1,0,1, respectively, when $u < 0, = 0, > 0$. The absolute value of ψ_n should be small when the process is IC, because the positive and negative values in the summation of (10) will be mostly canceled out in such cases. On the other hand, the value of ψ_n will be positively (negatively) large if there is an upward (downward) mean shift. Therefore, ψ_n can be used for detecting process mean shift. Also, it can be checked that the IC distribution of ψ_n does not depend on the original IC process distribution as long as that distribution is symmetric. In that sense, control charts constructed based on ψ_n are distribution-free. As a matter of fact, a number of distribution-free control charts based on ψ_n have been suggested in the literature. See, for instance, Bakir (2004), Chakraborti and Eryilmaz (2007), Graham et al. (2011), Li et al. (2010), and Mukherjee et al. (2013). For instance, the EWMA charting statistic suggested in Chapter 8 of Qiu (2014) is defined as

$$E_n = \lambda \psi_n + (1 - \lambda) E_{n-1}, \qquad \text{for } n \geq 1, \tag{11}$$

where $E_0 = 0$ and $\lambda \in (0, 1]$ is a weighting parameter. When the process distribution is symmetric, it can be checked that the IC mean and variance of E_n are 0 and $m(m+1)(2m+1)/6$, respectively. So, the chart gives a signal when

$$|E_n| > \rho_W \sqrt{\left[\frac{m(m+1)(2m+1)}{6}\right]\left[\frac{\lambda}{2-\lambda}\right][1-(1-\lambda)^{2n}]}, \tag{12}$$

where $\rho_W > 0$ is a parameter chosen to reach a given ARL_0 value.

Besides ψ_n, there are some alternative rank-based statistics used for constructing nonparametric control charts. These include the ones based on the sign test statistic (e.g., Lu 2015), the Cucconi test statistic (e.g., Chowdhury et al. 2014), the nonparametric likelihood ratio test (e.g., Zou and Tsung 2010), the Mann-Whitney two-sample test (e.g., Hawkins and Deng 2010), and more. For multivariate SPC problems, Qiu and Hawkins (2001, 2003) suggested CUSUM charts for detecting process mean shifts using antiranks of different quality variables, Zou and Tsung (2011) suggested an EWMA chart using spatial signs, and Zou et al. (2012) and Holland and Hawkins (2014) suggested different nonparametric control charts using spatial ranks. See Qiu (2018) for a discussion about other rank-based control charts.

The second type of nonparametric SPC charts is based on data categorization. In multivariate cases, the major difficulty in describing a process distribution when it is non-Gaussian is that the association among different quality variables can have infinitely many possibilities and it is hard to describe such association properly in general. However, if the quality variables are all categorical, then there are some mature statistical methodologies in the area of categorical data analysis (cf., Agresti 2002) for describing the association among categorical variables. Based on this observation, Qiu (2008) suggested a general framework for constructing nonparametric control charts based on data categorization and categorical data analysis. Assume that process observations are

$$\mathbf{X}_n = (X_{n1}, X_{n2}, \ldots, X_{np})', \qquad \text{for } n \geq 1,$$

where p is the dimension of the quality vector $\mathbf{X}$. Let the IC median of X_{nj} be $\widetilde{\mu}_j$, for $j = 1, 2, \ldots, p$, which can be estimated from an IC data. Define

$$Y_{nj} = I(X_{nj} > \widetilde{\mu}_j), \qquad \text{for } j = 1, 2, \ldots, p, \tag{13}$$

and $\mathbf{Y}_n = (Y_{n1}, Y_{n2}, \ldots, Y_{np})'$, where $I(a)$ is an indicator function that equals 1 if a is "true" and 0 otherwise. Then, $\mathbf{Y}_n$ is the categorized version of $\mathbf{X}_n$. The IC distribution of $\mathbf{Y}_n$, can be described by a log-linear model which can be estimated from the IC data. The estimated IC distribution of $\mathbf{Y}_n$ is denoted as

$$\left\{ f^{(0)}_{j_1 j_2 \cdots j_p}, \;\; j_1, j_2, \ldots, j_p = 1, 2 \right\},$$

where $j_1, j_2, \ldots, j_p$ are indices of the p-way contingency table associated with the categorized data $\mathbf{Y}_n$ and its distribution (note that each dimension of the contingency table has two categories). For $j_1, j_2, \ldots, j_p = 1, 2$, define

$$g_{n j_1 j_2 \cdots j_p} = I(Y_{n1} = j_1 - 1, Y_{n2} = j_2 - 1, \ldots, Y_{np}(i) = j_p - 1),$$

$\mathbf{g}_n$ is a vector of all $g_{n j_1 j_2 \cdots j_p}$ values, and $\mathbf{f}^{(0)}$ is the vector of all $f^{(0)}_{j_1 j_2 \cdots j_p}$. Then, $\mathbf{g}_n$ and $\mathbf{f}^{(0)}$ are vectors of the observed and expected counts of the contingency table at

the time point n, respectively. Let $\mathbf{U}_0^{obs} = \mathbf{U}_0^{exp} = \mathbf{0}$ be two 2^p-dimensional vectors, and

$$\begin{cases} \mathbf{U}_n^{obs} = \mathbf{U}_n^{exp} = \mathbf{0}, & \text{if } B_n \le k \\ \mathbf{U}_n^{obs} = \left(\mathbf{U}_{n-1}^{obs} + \mathbf{g}_n\right)(1 - k/B_n), & \text{if } B_n > k \\ \mathbf{U}_n^{exp} = \left(\mathbf{U}_{n-1}^{exp} + \mathbf{f}^{(0)}\right)(1 - k/B_n), \end{cases}$$

where

$$B_n = \left\{\left(\mathbf{U}_{n-1}^{obs} - \mathbf{U}_{n-1}^{exp}\right) + \left(\mathbf{g}_n - \mathbf{f}^{(0)}\right)\right\}' \left(\text{diag}(\mathbf{U}_{n-1}^{exp} + \mathbf{f}^{(0)})\right)^{-1} \left\{\left(\mathbf{U}_{n-1}^{obs} - \mathbf{U}_{n-1}^{exp}\right) + \left(\mathbf{g}_n - \mathbf{f}^{(0)}\right)\right\},$$

$k \ge 0$ is an allowance constant, $\text{diag}(\mathbf{a})$ denotes a diagonal matrix with its diagonal elements equal to the corresponding elements of the vector $\mathbf{a}$, and the superscripts "obs" and "exp" denote the observed and expected counts, respectively. Define

$$C_n = \left(\mathbf{U}_n^{obs} - \mathbf{U}_n^{exp}\right)' \left[\text{diag}(\mathbf{U}_n^{exp})\right]^{-1} \left(\mathbf{U}_n^{obs} - \mathbf{U}_n^{exp}\right). \tag{14}$$

Then, a location shift in $\mathbf{X}_n$ is signaled if

$$C_n > h, \tag{15}$$

where $h > 0$ is a control limit chosen to achieve a given ARL_0 level.

A chart similar to the one defined in (13)-(15) was suggested in univariate cases by Qiu and Li (2011a), where the number of categories in categorizing the quality variable can be larger than 2. From the large comparative studies in Qiu and Li (2011a,b), it can be seen that the chart based on data categorization has some advantages in terms of the ARL_1 metric over certain alternative nonparametric control charts in various cases considered, although it still has much room for possible improvement. For instance, data categorization would lose information in the original data. It is thus important to study how to make the lost information as small as possible while keeping the favorable properties of the related nonparametric control charts. In this regard, one possible improvement is to make use of the ordering information among the categories of the categorized data when constructing control charts, which has been discussed recently by Li (2017).

4 Charts for Monitoring Dynamic Processes

The conventional control charts discussed in Section 2 usually require the assumption that process observations have an identical distribution at different time points when the related process is IC. This assumption is not valid in certain applications. One example mentioned in Section 1 is about monitoring of disease incidence rates. In this example, the distribution of disease incidence rates would change over time

because of seasonality and other reasons, even in cases when there are no disease outbreaks. In such cases, the disease incidence rate process has a time-varying IC distribution. Such processes are called *dynamic* processes in this paper. For monitoring dynamic processes, the conventional control charts would be unreliable to use, and new monitoring charts are needed. Recently, we developed several control charts for monitoring dynamic processes, which are introduced below in this section.

Qiu and Xiang (2014) suggested a so-called *dynamic screening system (DySS)* for monitoring dynamic processes with a single quality/performance variable. The DySS method consists of three main steps described below.

Step I The regular longitudinal pattern of the performance variable y is first estimated from an *IC dataset* containing longitudinal observations of a group of m well-functioning subjects.

Step II For a new subject to monitor, his/her observations are first standardized using the estimated regular longitudinal pattern obtained in Step I.

Step III The standardized observations of the new subject are then monitored by a conventional control chart and a signal is given as soon as all available data suggest a significant shift in the longitudinal pattern of the subject under monitoring from the estimated regular pattern.

Assume that the observed longitudinal data of the m well-functioning subjects in the IC dataset follow the model

$$y(t_{ij}) = \mu(t_{ij}) + \sigma(t_{ij})\varepsilon(t_{ij}), \qquad \text{for } j = 1,2,\ldots,n_i,\ i = 1,2,\ldots,m, \tag{16}$$

where $t_{ij} \in [0,T]$ are observation times, $y(t_{ij})$ is the jth observation of the ith subject, $\mu(t_{ij})$ and $\sigma(t_{ij})$ are the mean and standard deviation of $y(t_{ij})$, and $\varepsilon(t_{ij})$ is the standardized random error with mean 0 and variance 1. Qiu and Xiang (2014) suggested a four-step procedure for estimating $\mu(t)$ and $\sigma^2(t)$ using the local linear kernel smoothing procedure. The estimators are denoted as $\widehat{\mu}(t)$ and $\widehat{\sigma}^2(t)$, respectively. For a subject to monitor, when his/her performance is IC, his/her observations should follow model (16), although the observation times, denoted as t_j^*, for $j = 1,2,\ldots$, could be different from those in model (1). So, his/her observations $\{y(t_j^*), j \geq 1\}$ can be standardized by

$$\widehat{\varepsilon}(t_j^*) = \frac{y(t_j^*) - \widehat{\mu}(t_j^*)}{\widehat{\sigma}(t_j^*)}, \qquad \text{for } j \geq 1, \tag{17}$$

where $\widehat{\mu}(t)$ and $\widehat{\sigma}(t)$ are estimated from the IC data. Obviously, the standardized observations $\{\widehat{\varepsilon}(t_j^*), j \geq 1\}$ in (17) would have mean 0 and variance 1 when the subject under monitoring is IC. To detect an upward mean shift in the original performance variable y for the given subject, Qiu and Xiang (2014) suggested using the CUSUM charting statistic defined as

$$C_j^+ = \max(0, C_{j-1}^+ + \widehat{\varepsilon}(t_j^*) - k), \qquad \text{for } j \geq 1, \tag{18}$$

where $C_0^+ = 0$ and $k > 0$ is an allowance constant. Then, the chart gives a signal when

$$C_j^+ > h_C, \tag{19}$$

where $h_C > 0$ is a control limit. For detecting a downward or arbitrary shift, a downward or two-sided CUSUM chart can be used.

As discussed in Section 2, the performance of a control chart is usually measured by ARL_0 and ARL_1. However, in many dynamic process monitoring applications, the observation times are often unequally spaced. In such cases, ARL_0 and ARL_1 are irrelevant any more. Instead, Qiu and Xiang (2014) suggested using the average time to signal (ATS), defined below. Let ω be a basic time unit so that all observation times are its integer multiples. Then, we define $n_j^* = t_j^*/\omega$, for $j = 1, 2, \ldots$, where $n_0^* = t_0^* = 0$. For a subject whose longitudinal performance is IC, assume that a signal is given at the sth observation time. Then, $E(n_s^*)$ is called the IC ATS, denoted as ATS_0. Similarly, for a subject whose longitudinal performance starts to deviate from the regular longitudinal pattern at the time point τ, the value $E(n_s^*|n_s^* \geq \tau) - \tau$ is called OC ATS, denoted as ATS_1. For the chart (18)-(19), the value of ATS_0 can be specified beforehand, and it performs better for detecting a shift of a given size if its ATS_1 value is smaller.

The DySS method discussed above is for monitoring univariate dynamic processes only. Its multivariate version has been developed in Qiu and Xiang (2015). Li and Qiu (2016, 2017) suggested univariate and multivariate DySS methods that were effective in cases when process observations were serially correlated. In the chart (18)-(19), the fact that process observations are often unequally spaced is considered in the performance measures ATS_0 and ATS_1 only, and it is not taken into account in the construction of the chart. This limitation was lifted in the EWMA chart proposed by Qiu et al. (2018).

In many applications, especially those outside the manufacturing industries, longitudinal processes for monitoring often have time-varying IC distributions. Therefore, dynamic process monitoring is an important research topic. Although we have developed a number of control charts for that purpose, there are still many open research problems. For instance, the DySS method described above depends heavily on the estimated regular longitudinal pattern obtained from an IC dataset. However, it is often challenging to choose an appropriate IC dataset in practice. Proper estimation of the regular longitudinal pattern is challenging as well, especially in cases when the size of the IC dataset is quite small.

5 Charts for Monitoring Spatio-Temporal Processes

We experienced SARS and many other damaging infectious diseases (cf., Fiore et al. 2010). To monitor their incidence rates, some global, national and regional reporting systems have been developed. For instance, Florida Department of Health (FDOH) has built the Electronic Surveillance System for the Early Notification of

Community-based Epidemics at Florida (ESSENCE-FL) recently, which is a syndromic surveillance system for collecting near real-time pre-diagnostic data from participating hospitals and urgent care centers in Florida. Figure 1 presents the observed incidence rates of influenza-like illness (ILI) for all 67 counties of Florida on 06/01/2012 (a summer time) and 12/01/2012 (a winter time). One important feature of these disease incidence data is that observed data collected at different places and different times are usually correlated: with the ones closer in space or time being more correlated. This kind of spatio-temporal (ST) correlation, however, is hidden in the observed data, and cannot be observed directly. Also, the disease incidence data have the seasonality and other temporal variation, and their temporal patterns could be different at different places (i.e., the spatial variation), as seen in Figure 1.

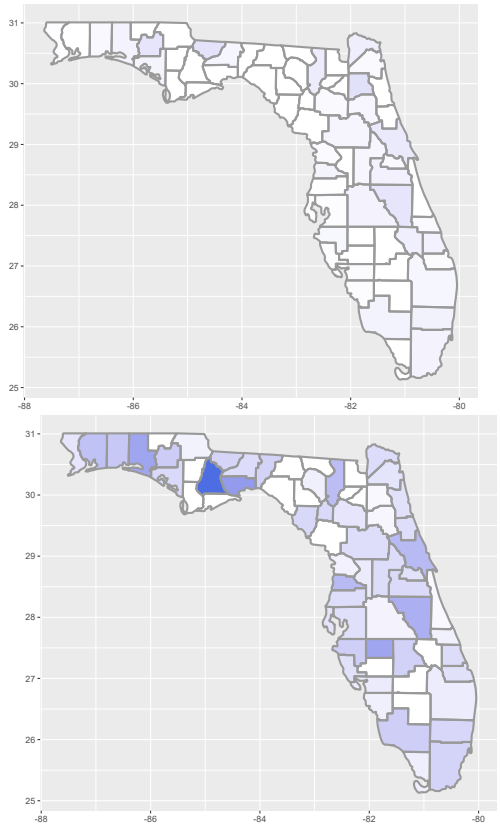

Fig. 1 Observed ILI incidence rates in Florida on 06/01/2012 (left) and 12/01/2012 (right). Stronger colors denote larger values.

The conventional SPC charts described in Section 2 require the assumptions that process observations are independent and identically distributed when the underlying process is IC. These assumptions are all violated in the ST process monitoring problem discussed above, because of the ST data correlation and the fact that disease

incidence rates would have time- and space-varying distribution even in cases when no disease outbreaks are present. Therefore, specific control charts for handling ST processes are needed.

In a case study for analyzing a foot, hand and mouth disease dataset, Zhang et al. (2015) suggested a procedure consisting of three steps: detrend, decorrelation, and sequential monitoring, which are briefly described below. (i) Seasonality in the observed disease incidence data is first described by a nonparametric longitudinal model, which can be estimated from an IC dataset and then eliminated from all observed data. (ii) Temporal autocorrelation in the detrended data is modeled by an ARIMA model, and then eliminated from the detrended data. (iii) The detrended and decorrelated data obtained in step (ii) are then sequentially monitored by an SPC chart. A similar procedure was used for analyzing an AIDS data in Zhang et al. (2016). This three-step method, however, can monitor the disease incidence rates at a single location only, and it cannot monitor the data at multiple locations simultaneously while accommodating ST data correlation properly.

To overcome the limitation of the three-step method by Zhang et al. (2015), Yang and Qiu (2018) suggested a flexible approach for spatio-temporal data modeling, briefly described below. Let Ω and $[0,T]$ be a 2-D region and a given time interval. The observed disease incidence rates in $\Omega \times [0,T]$ are assumed to follow the model

$$y(t_i,\mathbf{s}_{ij}) = \lambda(t_i,\mathbf{s}_{ij}) + \varepsilon(t_i,\mathbf{s}_{ij}), \qquad \text{for } j = 1,2,\ldots,m_i,\ i = 1,2,\ldots,n, \tag{20}$$

where $t_i \in [0,T]$ is the ith observation time point, $\mathbf{s}_{ij} \in \Omega$ is the jth observation location at time t_i, $\varepsilon(t_i,\mathbf{s}_{ij})$ is a zero-mean random error, m_i is the number of observation locations at time t_i, and n is the number of time points in the dataset. The correlation structure in the observed data can be described by the covariance function

$$V(\boldsymbol{u};\boldsymbol{v}) = E\left[\varepsilon(\boldsymbol{u})\varepsilon(\boldsymbol{v})\right] = \sigma(\boldsymbol{u})\sigma(\boldsymbol{v})Cor(\varepsilon(\boldsymbol{u}),\varepsilon(\boldsymbol{v})), \qquad \text{for } \boldsymbol{u},\boldsymbol{v} \in [0,T]\times\Omega, \tag{21}$$

where $\sigma^2(\cdot)$ is the variance function and $Cor(\cdot,\cdot)$ is the correlation function. The mean function $\lambda(t,\mathbf{s})$ is then estimated by a spatio-temporal local linear kernel smoothing procedure. The estimator is denoted as $\widehat{\lambda}(t,\mathbf{s})$. To accommodate the ST data correlation, the bandwidths used in estimating $\lambda(t,\mathbf{s})$ should be chosen carefully. To this end, a modified cross-validation procedure was proposed in Yang and Qiu (2018). After $\widehat{\lambda}(t,\mathbf{s})$ is obtained, $V(\boldsymbol{u},\boldsymbol{v})$ can be estimated by moment estimation from the residuals. The resulting estimator is denoted as $\widehat{V}(\boldsymbol{u},\boldsymbol{v})$.

Based on the above ST data modeling approach, Yang and Qiu (2019) suggested a CUSUM chart for monitoring ST processes, which consists of several steps. First, the regular longitudinal pattern of the spatial disease incidence rates in cases when no disease outbreaks are present can be described by $\lambda(t,\mathbf{s})$ and $V(\boldsymbol{u},\boldsymbol{v})$ in (20) and (21), which can be estimated from an IC dataset by the ST modeling procedure discussed above. Then, they can be used for online monitoring of the disease incidence rates $y(t_i^*,\mathbf{s}_{ij}^*)$ observed at locations $\{\mathbf{s}_{ij}^*, j = 1,2,\ldots,m_i^*\}$ and times t_i^*, for $i = 1,2,\ldots$. Define $\boldsymbol{y}(t_i^*) = (y(t_i^*,\mathbf{s}_{i1}^*), y(t_i^*,\mathbf{s}_{i2}^*),\ldots,y(t_i^*,\mathbf{s}_{im_i^*}^*))'$, for all i. When the process is IC, the observed data are assumed to follow model (20) in the sense

that $y(t_i^*, \mathbf{s}_{ij}^*) = \lambda(t_i^*, \mathbf{s}_{ij}^*) + \varepsilon(t_i^*, \mathbf{s}_{ij}^*)$, for $j = 1, 2, \ldots, m_i^*$ and $i = 1, 2, \ldots$, and the mean function $\lambda(t, \mathbf{s})$ is assumed periodic in time with the period T. Namely, $\lambda(t_i^*, \mathbf{s}_{ij}^*) = \lambda(t_i^{**}, \mathbf{s}_{ij}^*)$, where $t_i^* = t_i^{**} + \ell T$ for all i, $t_i^{**} \in [0, T]$, and $\ell \geq 1$ is an integer. Second, decorrelate and standardize all observed data up to the current time point i: $\{\boldsymbol{y}(t_1^*), \boldsymbol{y}(t_2^*), \ldots, \boldsymbol{y}(t_i^*)\}$. The decorrelated and standardized data are denoted as $\widehat{\widetilde{\boldsymbol{e}}}(t_1^*), \widehat{\widetilde{\boldsymbol{e}}}(t_2^*), \ldots, \widehat{\widetilde{\boldsymbol{e}}}(t_i^*)$. Then, the suggested CUSUM charting statistic for detecting upward shifts in the disease incidence rates is

$$C_i^+ = \max\left(0, C_{i-1}^+ + \frac{\widehat{\widetilde{\boldsymbol{e}}}(t_i^*)' \widehat{\widetilde{\boldsymbol{e}}}(t_i^*) - m_i^*}{\sqrt{2m_i^*}} - k\right), \qquad \text{for } i \geq 1, \tag{22}$$

where $C_0^+ = 0$ and $k > 0$ is an allowance constant. The chart gives a signal when

$$C_i^+ > \gamma, \tag{23}$$

where $\gamma > 0$ is a control limit. To determine γ in (23) so that the chart (22)-(23) has a specific ARL_0 value, Yang and Qiu (2019) suggested using a block bootstrap procedure.

6 Concluding Remarks

In the previous sections, we have briefly introduced some recent research in SPC, after an introduction of four types of basic SPC charts. The recent research introduced is mainly on nonparametric SPC, dynamic process control, and spatio-temporal process monitoring. These research topics aim to handle cases when the regular assumptions in SPC that IC process observations are independent and identically distributed with a specific parametric distribution are violated. In each of these research topics, there are still many open questions that need to be addressed in our future research. For instance, in the nonparametric SPC area, there have been many nonparametric control charts proposed. Systematic comparison of these charts should be important for them to be used in real-data applications. Also, both ranking and data categorization would lose information in the original observed data. It should be important to study how to minimize the lost information while keep all the favorable properties of nonparametric control charts. For dynamic process monitoring, accurate estimation of the regular longitudinal pattern from the IC data is critically important. Usually, the size of IC data is limited. In such cases, self-starting procedures to expland the initial IC dataset, by combining it with observed data during procedure monitoring after it is confirmed that the process under monitoring is IC at the current time point, might be one way to overcome the difficulty, which needs to be further studied in the future. Proper monitoring of spatio-temporal processes is important but challenging. The chart (22)-(23) represents our first research effort on that topic, and many issues, including proper accommodation of important covariates, need to be addressed in future research.

In the big data era, SPC will find more and more applications (cf., Qiu 2017, 2018b). In these new applications, the related process monitoring problems could become more complicated. For instance, sequential monitoring of images has broad applications in manufacturing industries, traffic monitoring, medical diagnostics, and more. But, images often have edges and other complicated structures. Also, images obtained at different times should be geometrically aligned properly for meaning analysis of the image sequence. These features of image data, however, would make proper monitoring of an image sequence extremely challenging. So, new SPC methods are needed for handling such new applications.

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