ACCELERATED LIFE TESTING MODEL BUILDING WITH BOX-COX TRANSFORMATION

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Abstract

In accelerated life testing, the nominal life time is often related to stress levels by an acceleration equation. Three particular models that have been used frequently in the past are the power law model, the Arrhenius model and the Eyring model. In this paper we suggest choosing a model from a model family which includes the three particular models as special cases. This family is defined by a Box-Cox transformation on the stress variable. There are two benefits to use this proposal: (1) model fitting could be treated in an unified way; and (2) the fitted model is more robust to model assumptions. We demonstrate this method with some numerical examples.

Key Words: Accelerated life testing, Acceleration equation, Box-Cox transformation, Residual sum of squares.

1 Introduction

In accelerated life testing, products are tested under higher than usual levels of stresses to shorten the testing time and to get more failures (Nelson 1990; Meeker and Escobar 1993). To estimate life times at normal stress levels based on the accelerated life testing data is a process of extrapolation. This process is often accomplished by using a predetermined acceleration equation which relates the life time of products to the stress levels. Three particular acceleration equations that have been used frequently in the past are the power law model, the Arrhenius model and the Eyring model (Levenbach 1957; Thomas 1964).

The power law model has a linear expression: $\mu = a + b[-\log(S)]$, where $\mu = log(\eta)$, η is the nominal life time (some parameter of the life time distribution), S denotes the stress variable, a and b are the coefficients. This model is often used to relate product life to pressure-like stresses (e.g., voltage). It is generally viewed as being an empirical model, but with large amounts of experimental verification (see a list of applications of this model in Section 2.10, Nelson 1990). The Arrhenius life relationship, $\mu = a + b/S$, is widely used to model product life as a function of temperature. Applications include electrical insulations and dielectrics, battery cells, plastics, etc. It is a first-order approximation to the following Eyring model: $\mu = \log(A) - \log(S) + B/S$, where A and B are constants. In most applications, A/S is essentially constant due to the small range of temperature, making the Eyring model close to the Arrhenius relationship.

The above three models have the following common structure:

$$\mu = a + b\phi(S), \tag{1.1}$$

where $\phi(\cdot)$ is some prespecified function, a and b are the coefficients. In the literature (e.g., Chapters 4 and 5, Nelson 1990), the model coefficients are often estimated by the least squares (LS) method and the maximum likelihood estimation (MLE) method. By using the LS method, μ needs to be estimated from the experimental data first at each stress level and then model (1.1) is fitted in the usual way.

Model (1.1) is based on two assumptions: (1) the function $\phi(\cdot)$ needs to be completely specified, and (2) the relationship between μ and $\phi(S)$ is linear. If one of these two assumptions is violated in a specific application, then results from the extrapolation procedure will not be reliable. Therefore it is emphasized in the literature (e.g., Chapter 2, Nelson 1990; Chapter 18, Meeker and Escobar 1998) to fully understand the mechanism of the application problems such that appropriate models could be identified for extrapolation. It is also emphasized to verify the empirical models over the entire range of the stress variables. But it might not be easy to do so in some cases because the life time of some products could be extremely long under low stresses. In this paper, we make an attempt to try to partially overcome this difficulty by considering a more flexible model.

Figure 1.1 demonstrates four possible cases. The stress S in these cases is the temperature T. We consider seven T levels: $150^oC, 200^oC, 250^oC, 300^oC, 350^oC, 400^oC$ and 450^oC. In plot (a), the true relationship between μ and $\phi(T)$ is $\mu = 38.3 + 5\phi(T) + 5log(\phi(T))$ where $\phi(T) = 1/T$. The lineality assumption of (1.1) is violated in this case. (It is an Arrhenius model if the term

$5log(\phi(T))$ does not exist.) The "+" points in Figure 1.1(a) denote $\{(\phi(T_i), \hat{\mu}_i)\}$ where $\hat{\mu}_i$ is an estimator of μ at stress levels T_i. The dotted curve represents the true regression model. The dashed line is the fitted LS line by using the Arrhenius model. The solid curve denotes the fitted model by our proposal. It can be seen that all three curves/lines are close to each other in the design range (T between 150^oC and 450^oC). But when they are used for extrapolation at normal temperature levels (say, $T \leq 100^oC$), their difference is obvious. Extrapolation results from our proposal are close to the truth while those from the Arrhenius model are far away from the truth. This example shows that the extrapolation procedure is sensitive to the lineality assumption if the Arrhenius model is used in statistical analysis and our proposal partially overcomes this problem.

Plot (b) demonstrates another case that the true relationship between μ and $\phi(T)$ is linear. But $\phi(T)$ equals to $log(T)/T^{0.6}$ instead of $1/T$. The true model is $\mu = 3.7 + 18.5\phi(T)$. We plot $\{(\phi(T_i), \hat{\mu}_i)\}$ by "+" points as before. The dotted straight line is the true regression model. The dashed curve represents the fitted Arrhenius model (it does not appear to be straight in plot (b) because the scale used for the x-axis is by $log(T)/T^{0.6}$ instead of by T). The solid curve is the fitted model by our proposal. The extrapolation results by using the Arrhenius model do not look good in this case either. On the other hand, our method still behaves reasonably well.

Plots (c) and (d) show another two cases related to the power law model. The true relationship between μ and T is $\mu = 20.3 + 1500/T - 3log(T)$ in plot (c); and $\mu = 475 - 195log(T)/T^{0.15}$ in plot (d). More explanation about these plots could be found in Section 3.2.

There are two possible approaches to make the model (1.1) more flexible. The first approach is to allow for nonlineality in relating μ and $\phi(S)$ and the second one is to make the function specification for $\phi(S)$ more flexible. We choose the second stratedy in this paper. Instead of being fully specified, $\phi(S)$ belongs to a function family in our proposal. This family is defined by the following Box-Cox transformation (Box and Cox 1964):

$$BC_\lambda(S) := \begin{cases} \frac{S^\lambda - 1}{\lambda}, & \text{if } \lambda \neq 0 \\ log(S), & \text{otherwise} \end{cases} \tag{1.2}$$

where λ is a function index. The Box-Cox transformation is commonly used in regression (see e.g., Section 5.3, Draper and Smith 1981). When $\lambda = 0$ and $\lambda = -1$, model (1.2) corresponds to the power law model and the Arrhenius model, respectively. In Section 3, we will show that the Eyring model could be approximated well by some member in this family. Therefore the function family

$\{BC_\lambda(S), \lambda \in (-\infty, \infty)\}$ is large enough to cover most models used in the accelerated life testing literature.

In recent years statisticians have made a great effort in developing flexible models for accelerated life testing. Nonparametric methods (Basu and Ebrahimi 1982; Schmoyer 1991) can avoid some difficulties in model specification. But it is often hard to measure their goodness of fit because the number of stress levels is limited in most situations. Our model is essentially a compromise between model (1.1) and the nonparametric models. It assigns one degree of freedom to $\phi(S)$ by using the function index λ. Other methods in this direction include the life tests planning with a nonconstant scale parameter (Kvam and Samaniego 1993; Meeter and Meeker 1994), planning the life tests with experimental design techniques (McKinney 1993), etc.

In model (1.2), the function index λ could be chosen by minimizing the residual sum of squares (RSS). The entire model building procedure will be introduced in Section 2. In Section 3, some simulation results are presented to compare our model with several existing models. We also apply the new method to a published data set. Several remarks conclude the article in Section 4.

2 Model Building With Box-Cox Transformation

As mentioned in Section 1, both LS and MLE methods could be used in estimating the unknown parameters in (1.1) and (1.2). For simplicity, only the model building process with LS estimation is demonstrated here, which consists of two steps: (i) estimating μ from experimental data at each stress level; and (ii) estimating coefficients a and b of model (1.1) where $\phi(S)$ belongs to the function family (1.2).

2.1 Estimate μ

In accelerated life testing, it is often assumed that products' lifetime distribution belongs to a given parametric distribution family (e.g., Exponential, Gamma, Lognormal or Weibull distribution families). One parameter of the lifetime distribution (which is related to the mean lifetime) is often related to the stress variable by model (1.1) while all other distribution parameters are assumed to be constants. In model (1.1), μ needs to be estimated from the experimental data at each stress level. Many procedures have been developed in the literature to estimate μ under various distribution assumptions and for several different types of censored data (cf. e.g., Bain and Engelhardt 1991; Lawless 1983). For example, if the lifetime distribution is assumed to be a Weibull distribution and the data is under type II censoring, then the Best Linear Unbiased Estimator (BLUE) of μ can be obtained as follows (Nelson 1982).

Suppose that the lifetime T has the following two-parameter Weibull distribution.

$$F_T(t) = 1 - exp\{-(t/\eta)^m\},\ t > 0, \tag{2.1}$$

where m and η are the shape and scale parameters, respectively. At a given stress level, n products are assumed to be tested. The experiment ends after r failures are observed and $T_1 \leq T_2 \leq \cdots \leq T_r$ denote the first r failure times. Let $X_i := log(T_i), Y_i := (X_i - \mu)/\sigma, E(Y_i) = \alpha_i$ and $cov(Y_i, Y_j) = \nu_{ij}$, for $i, j = 1, 2, \cdots, r$, where $\mu = log(\eta)$ and $\sigma = 1/m$. Then

$$E(X_i) = \mu + \sigma\alpha_i,\ i = 1, 2, \cdots, r. \tag{2.2}$$

Equation (2.2) could be regarded as a linear regression equation and its parameters μ and σ could be estimated by the Gauss-Markov Theorem as follows.

$$\begin{pmatrix} \hat{\mu} \\ \hat{\sigma} \end{pmatrix} = (M'V^{-1}M)^{-1}M'V^{-1}\underline{X} \tag{2.3}$$

where

$$M = \begin{pmatrix} 1 & \alpha_1 \\ 1 & \alpha_2 \\ \vdots & \vdots \\ 1 & \alpha_r \end{pmatrix},\ V = \begin{pmatrix} \nu_{11} & \nu_{12} & \cdots & \nu_{1r} \\ \nu_{21} & \nu_{22} & \cdots & \nu_{2r} \\ \cdots & \cdots & & \cdots \\ \nu_{r1} & \nu_{r2} & \cdots & \nu_{rr} \end{pmatrix},\ \underline{X} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_r \end{pmatrix}.$$

In applications, (2.3) could be replaced by

$$\hat{\mu} = \sum_{i=1}^{r} D(n,r,i)X_i,\ \ \hat{\sigma} = \sum_{i=1}^{r} C(n,r,i)X_i,\ \ Var(\hat{\mu}) = \sigma^2 A_{r,n},\ \ Var(\hat{\sigma}) = \sigma^2 B_{r,n} \tag{2.4}$$

where $D(n,r,i), C(n,r,i), A_{r,n}$ and $B_{r,n}$ are constants and their values can be found from some specific tables (see e.g., Appendices 12a and 12b, Nelson 1982).

2.2 Fit model (1.1)

Suppose that we have k stress levels: $S_1, S_2, \cdots, S_k$. At stress level S_i, r_i failures are observed from n_i products. The nominal lifetime μ is estimated as in Section 2.1 at each stress level. The

Box-Cox transformation (1.2) is used for $\phi(S)$. Then model (1.1) can be expressed in the following matrix form (see also (2.4)):

$$\begin{cases} E(\underline{\hat{\mu}}) = \Phi_\lambda \underline{\theta} \\ Var(\underline{\hat{\mu}}) = \sigma^2 A \end{cases} \tag{2.5}$$

where

$$\underline{\hat{\mu}} = \begin{pmatrix} \hat{\mu}_1 \\ \hat{\mu}_2 \\ \vdots \\ \hat{\mu}_k \end{pmatrix}, \ \Phi_\lambda = \begin{pmatrix} 1 & BC_\lambda(S_1) \\ 1 & BC_\lambda(S_2) \\ \vdots & \vdots \\ 1 & BC_\lambda(S_k) \end{pmatrix}, \ \underline{\theta} = \begin{pmatrix} a \\ b \end{pmatrix}, \ A = \begin{pmatrix} A_{r_1,n_1} & & & 0 \\ & A_{r_2,n_2} & & \\ & & \ddots & \\ 0 & & & A_{r_k,n_k} \end{pmatrix}.$$

Similar to equation (2.2), (2.5) could also be regarded as a linear regression equation and thus $\underline{\theta}$ could be estimated by

$$\underline{\hat{\theta}} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (\Phi_\lambda' A^{-1} \Phi_\lambda)^{-1} \Phi_\lambda' A^{-1} \underline{\hat{\mu}}.$$

After some algebraic manipulations, we have

$$\hat{a} = \frac{GJ - IM}{EG - I^2}, \ \hat{b} = \frac{EM - IJ}{EG - I^2}, \tag{2.6}$$

where $E = \sum_{i=1}^k A_{r_i,n_i}^{-1}$, $I = \sum_{i=1}^k A_{r_i,n_i}^{-1} BC_\lambda(S_i)$, $G = \sum_{i=1}^k A_{r_i,n_i}^{-1} BC_\lambda(S_i)^2$, $J = \sum_{i=1}^k A_{r_i,n_i}^{-1} \hat{\mu}_i$ and $M = \sum_{i=1}^k A_{r_i,n_i}^{-1} BC_\lambda(S_i) \hat{\mu}_i$.

Since $\hat{\mu}_1, \hat{\mu}_2, \cdots, \hat{\mu}_k$ are estimated from k different samples at k stress levels, they are independent of each other. They are approximately normally distributed when $r_1, r_2, \cdots, r_k$ are large (cf. e.g., Section 7.4, Nelson 1982). Consequently, both $\hat{a}$ and $\hat{b}$ are approximately normally distributed in such case. At a specific stress level S_0, the estimated value of μ is

$$\hat{\mu}(S_0) = \hat{a} + \hat{b} BC_\lambda(S_0)$$

with variance

$$Var(\hat{\mu}(S_0)) = \sigma^2 (1, BC_\lambda(S_0)) (\Phi_\lambda' A^{-1} \Phi_\lambda)^{-1} (1, BC_\lambda(S_0))'.$$

Therefore a $100(1-\alpha)\%$ confidence interval for μ in the case when $r_1, r_2, \cdots, r_k$ are large is:

$$\hat{\mu}(S_0) \pm t_{\alpha/2,k-1} s(\hat{\mu}(S_0)) \tag{2.7}$$

where $t_{\alpha/2,k-1}$ is the $1 - \alpha/2$ quantile of the t-distribution with degrees of freedom $k-1$ and $s(\hat{\mu}(S_0))$ is the standard deviation of $\hat{\mu}(S_0)$ with σ replaced by an estimate $\hat{\sigma}$. Consequently when

$r_1, r_2, \cdots, r_k$ are large, a $100(1-\alpha)\%$ confidence interval for η, the nominal life time, is:

$$(\log^{-1}(\hat{\mu}(S_0) - t_{\alpha/2,k-1}s(\hat{\mu}(S_0))), \log^{-1}(\hat{\mu}(S_0) + t_{\alpha/2,k-1}s(\hat{\mu}(S_0)))). \tag{2.8}$$

In linear regression analysis, it has been shown that the confidence interval formula (2.7) is quite robust to the normal distribution assumption on the responses ($\hat{\mu}_1, \hat{\mu}_2, \cdots, \hat{\mu}_k$ could be regarded as responses in model (2.5)) partly because of the central limit theorem (cf. e.g., Chapter 6, Seber 1977). So formula (2.8) should still work well when some $\{r_i\}_{i=1}^k$ are small. A numerical example is presented in Section 3 to further discuss this issue.

3 Numerical Analysis

3.1 Approximate the Eyring model

The Eyring model can be expressed by

$$\mu = log(A) - log(T) + \frac{B}{T}, \tag{3.1}$$

where A and B are constants depending on the product and the test method. Next we demonstrate that model (3.1) could be approximated well by some function in the function family (1.2). That is equivalent to saying that the three most popular models (the power law model, the Arrhenius model and the Eyring model) could be treated in an unified way by using the Box-Cox transformation.

The right-hand side of (3.1) is a linear combination of two functions $log(T)$ and $1/T$. Both of them belong to the function family (1.2). Their weights are controled by the value of B: the bigger, the closer to the function $1/T$, and vice versa. Let $T \in [50^0C, 450^0C]$, $log(A) = 8.3$, and B vary from 0 to 5000. The best λ to approximate (3.1) is plotted in Figure 3.1(a). It varies from 0 to -0.97. MSE values of the best approximations are in the interval $[2.8 \times 10^{-6}, 2.55 \times 10^{-3}]$. The ratio of the MSE value to the square value of the range of function $log(A) - log(T) + B/T$ (we call this ratio the relative MSE) is plotted in Figure 3.1(b). It can be seen that both MSE and relative MSE values are small. We plot four intermediate cases with $B = 10$, 100, 250 and 1000 in plots (c)-(f), respectively. These plots show that difference between the Eyring model and its approximation is negligible.

3.2 Compare different models

In Figure 1.1(a), 12 observations are generated at each stress level from the Weibull distribution (2.1) with $m = 2$ and $log(\eta) = \mu = 38.3 + 5/T - 5log(T)$. These values are then ordered from the smallest to the largest. The first 9 ordered numbers (corresponding to a type II censoring with $r = 9$) are used to get the BLUE $\hat{\mu}$ of μ (see (2.4)). Based on $\{(T_i, \hat{\mu}_i)\}$, model (1.1) is fitted by using the Arrhenius model and the Box-Cox transformation method, respectively. We then extrapolate the fitted models at lower temperature levels. This whole process is repeated 100 times. The averaged predicted values are plotted by the dashed curve for the Arrhenius model and by the solid curve for the Box-Cox transformation method. One realization of $\{(1/T_i, \hat{\mu}_i)\}$ is plotted by the "+" points. Plots (b), (c) and (d) are generated in a similar way.

Figure 1.1 shows that when one of the two assumptions (namely, "linearlity" and "$\phi(S)$ is known") of model (1.1) is violated, extrapolation is risky based on either the Arrhenius model or the power law model. The Box-Cox transformation method, however, still provides a reasonable prediction. The mechanism behind the latter method is that when one assumption is violated, the Box-Cox transformation will adjust the other assumption to make the entire model to fit the data well.

Next we extend the case of Figure 1.1(a) in the following way. Suppose that the true relationship between μ and T is $\mu = 38.5 + 5/T - 5a[log(T)]$ where $a \geq 0$ is a constant. The case of Figure 1.1(a) corresponds to $a = 1$. This model is closer to an Arrhenius model when a is smaller. Figure 3.2 shows the extrapolation results from both the Box-Cox transformation method and the Arrhenius model. Plots (a)-(d) present the results at $T = 60^oC, 80^oC, 100^oC$ and 120^oC, respectively. From the plots, we can see that (1) results from the Box-Cox transformation method are closer to the truth than those from the Arrhenius model; (2) when a is bigger (the true model is more different from the Arrhenius model), the benefit to use the Box-Cox transformation method is more obvious; (3) the extrapolation procedure performs better when T increases. Similar results can be obtained for the cases of Figures 1.1(b)-(d).

Table 3.1: Coverage rates of the 95% confidence interval of η generated by formula (2.8) at seven T levels.

T level	150^oC	200^oC	250^oC	300^oC	350^oC	400^oC	450^oC
true η value	101.008	87.366	79.329	73.981	70.138	67.228	64.937
coverage rate	0.948	0.950	0.949	0.953	0.957	0.946	0.940

3.3 Performance of the confidence interval formula (2.8) when some $\{r_i\}_{i=1}^k$ are small

We now use the following example to evaluate the performance of the confidence interval formula (2.8) in the case when some $\{r_i\}_{i=1}^k$ are small. Assume that the true relationship between μ and T is $\mu = 3.7 + 18.5/T^{0.6}$ which belongs to the function family (1.2). As in Figure 1.1, seven T levels: $150^oC, 200^oC, 250^oC, 300^oC, 350^oC, 400^oC$ and 450^oC are considered. At each T level, 12 observations are generated from the Weibull distribution (2.1) with $m = 2$ and $\log(\eta) = \mu$. Then the 9 smallest observations are used to estimate μ. Therefore $k = 7$, $n_i = 12$, $r_i = 9$, for $i = 1, 2, \cdots, 7$, in this example. At each T level, 1000 95% confidence intervals of η are generated with formula (2.8) by repeating the simulation process 1000 times. The number of such intervals which cover the true value of η is then counted, defining the coverage rate of the confidence interval formula. The results are presented in Table 3.1. It can be seen that all coverage rates at seven T levels are quite close to the nominal rate 0.95 although $\{r_i\}_{i=1}^k$ are small in this example.

3.4 An application

In this part, we apply the Box-Cox transformation method to a pubilshed data set from Nelson (1970). The data (presented in Table 3.2) are breakdown times (in minutes) for seven groups of specimens, each group involving a different voltage level. The data are uncensored. The distribution of the breakdown time is assumed to be Weibull. Regular hypothesis testing procedure shows that the values of the shape parameter at different voltage levels are not significantly different (Example 4.3.2, Lawless 1982). The MLE estimates of the scale parameter at different voltage levels have been provided by several authors. Those provided by Singpurwalla and Al-Khayyal (1977) are used in this paper, which are plotted in Figure 3.3 in log scale by the "+" points. The MLE estimate of the shape parameter is 0.81. If we apply the power law model to this data set as several authors did, then the fitted model is plotted by the dashed line. The solid curve represents the fitted model by the Box-Cox transformation method. Some results are summarized in Table 3.3. From the MSE

Table 3.2: The breakdown time data from Nelson (1970).

Voltage Level	n_i	Breakdown Times
26	3	5.79, 1579.52, 2323.7
28	5	68.85, 426.07, 110.29, 108.29, 1067.6
30	11	17.05, 22.66, 21.02, 175.88, 139.07, 144.12, 20.46, 43.4, 194.9, 47.3, 7.74
32	15	0.4, 82.85, 9.88, 89.29, 215.1, 2.75, 0.79, 15.93, 3.91, 0.27, 0.69, 100.58, 27.8, 13.95, 53.24
34	19	0.96, 4.15, 0.19, 0.78, 8.01, 31.75, 7.35, 6.5, 8.27, 33.91, 32.52, 3.16, 4.85, 2.78, 4.67, 1.31, 12.06, 36.71, 72.89
36	15	1.97, 0.59, 2.58, 1.69, 2.71, 25.5, 0.35, 0.99, 3.99, 3.67, 2.07, 0.96, 5.35, 2.9, 13.77
38	8	0.47, 0.73, 1.4, 0.74, 0.39, 1.13, 0.09, 2.38

Table 3.3: Summary of the results by the power law model and the Box-Cox transformation method.

Methods	Fitted Model	MSE	predicted scale parameter at $V = 20$ and its 95% confidence interval
Power Law	$\mu = 64.518 - 17.655 log(V)$	0.258	112283.023 (1190.421,10590819)
Box-Cox	$\mu = 26.15 - 1.47\frac{V^{0.72}-1}{0.72}$	0.206	37794.623 (704.519,2027523)

values, our model fits the data better. It can be seen that the extrapolation results from these two methods are quite different. The predicted value of the scale parameter from the fitted power law model at voltage level $20kV$ is about 3 times the predicted value from our method. It can also be noticed that the confidence intervals from both methods are wide. Therefore it is important to have low stress testing (when it is possible) to reduce reliance on extrapolation.

4 Concluding Remarks

We have presented a model building procedure for accelerated life testing which is based on a function family defined by the Box-Cox transformation. Relationship between the nominal lifetime and the stress variable is built in an unified way by this procedure. Numerical examples show that extrapolation results from this procedure often outperform those from the three acceleration equations that are used frequently in the literature: the power law model, the Arrhenius model and the Eyring model.

As mentioned at the end of Section 1, the Box-Cox transformation procedure is a trade-off between linear and nonparametric acceleration models by assigning one degree of freedom to $\phi(S)$. It might be possible to assign more than one degree of freedom to $\phi(S)$ to make the model even more flexible. The base function of the Box-Cox transformation is the power function. It might be interesting to define a function family similar to (1.2) by using some more flexible base functions. It is not clear at this moment how to generalize the Box-Cox transformation procedure to the case with more than one stress variable. All these problems could be the future research topics.

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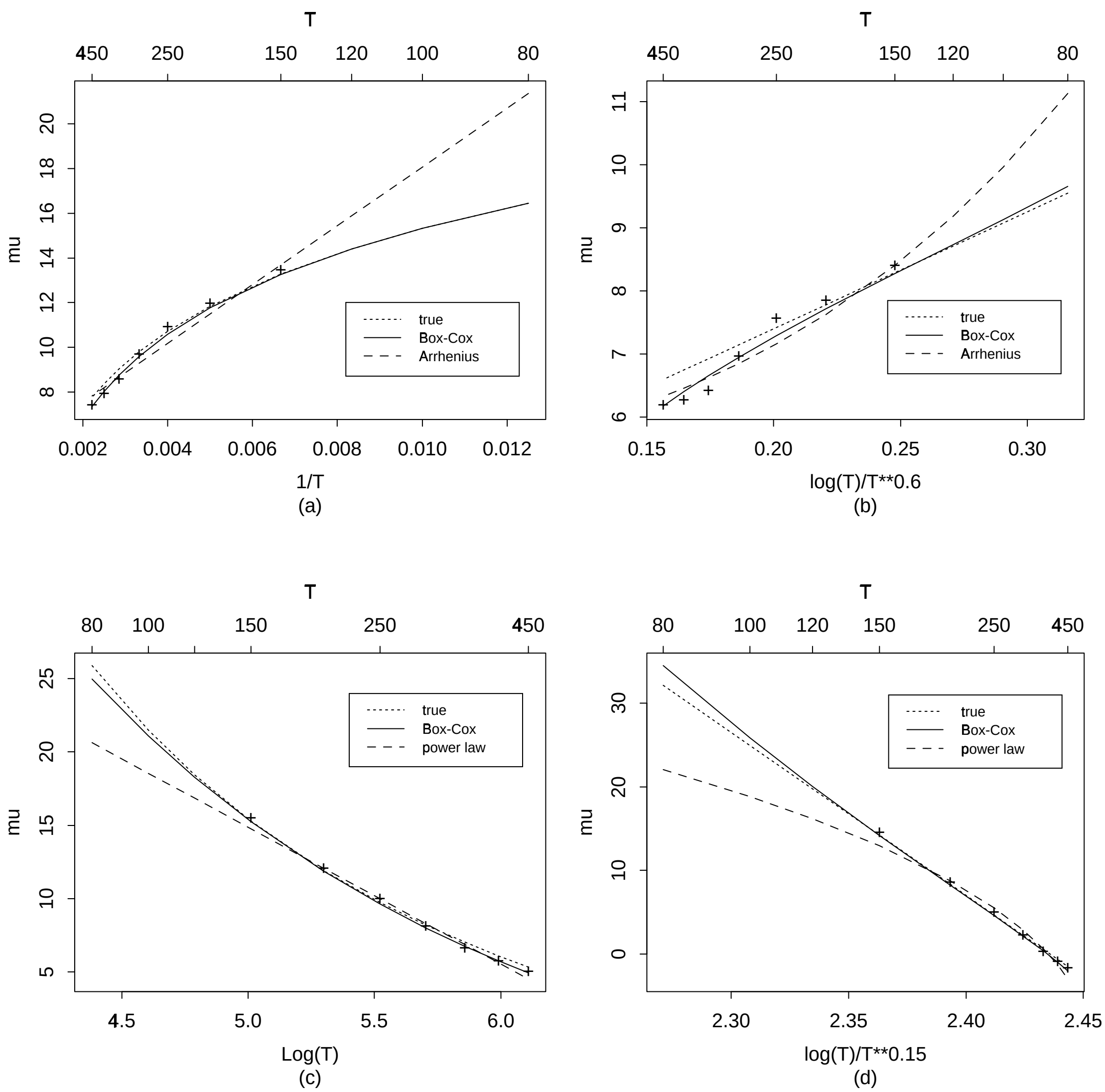

Figure 1.1: The "+" points are $\{(1/T_i, \hat{\mu}_i)\}$ in plot (a); $\{(log(T_i)/T_i^{0.6}, \hat{\mu}_i)\}$ in plot (b); $\{(log(T_i), \hat{\mu}_i)\}$ in plot (c); and $\{(log(T_i)/T_i^{0.15}, \hat{\mu}_i)\}$ in plot (d). The dotted curves represent the true models. The solid curves denote the fitted models by our proposal. The dashed curves are the fitted models by using the Arrhenius model in plots (a) and (b); by using the power law model in plots (c) and (d).

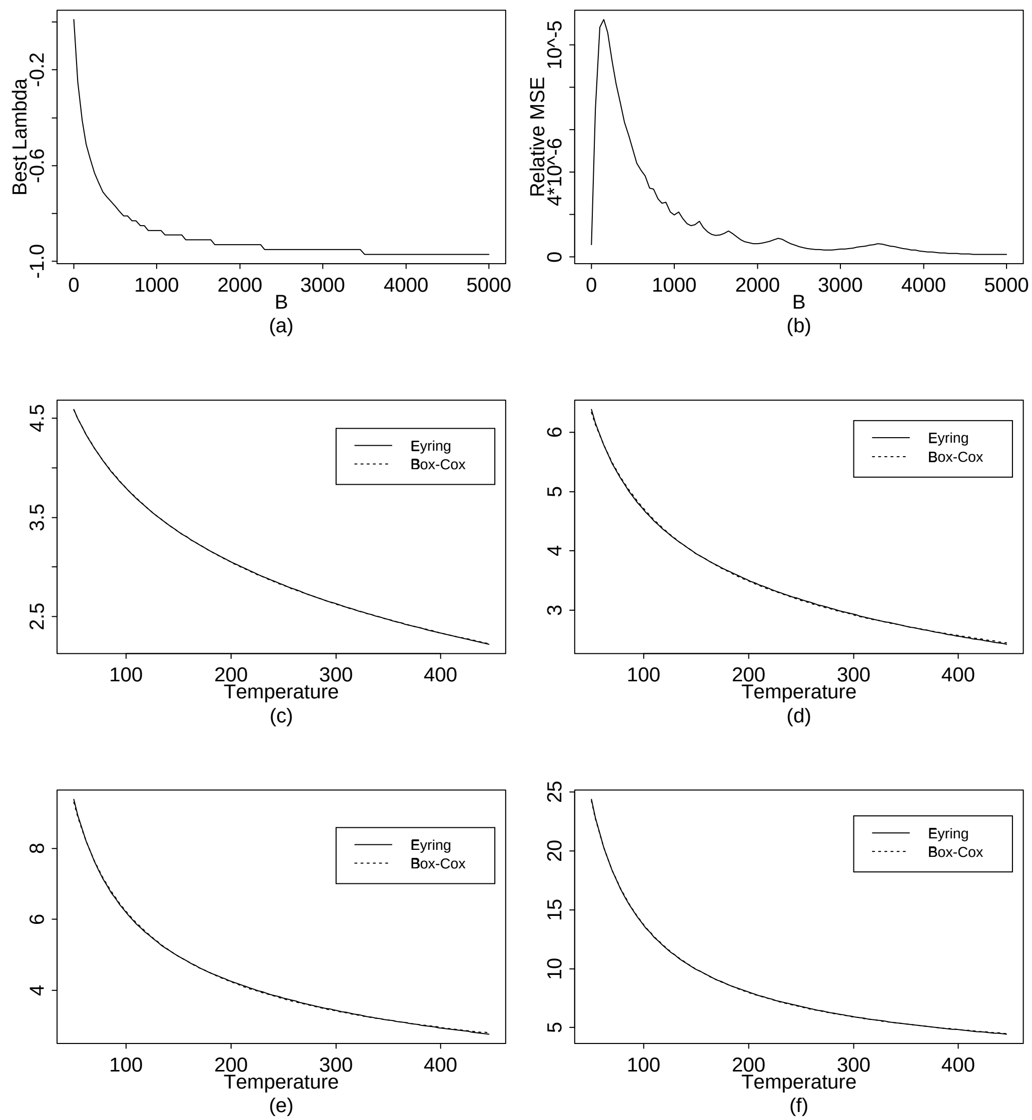

Figure 3.1: Approximate the Eyring model by the Box-Cox transformation. (a) Best λ versus B. (b) Relative MSE versus B. (c)-(f) The Eyring function and its best approximation by the Box-Cox transformation. (c) $B = 10$; (d) $B = 100$; (e) $B = 250$; (f) $B = 1000$.

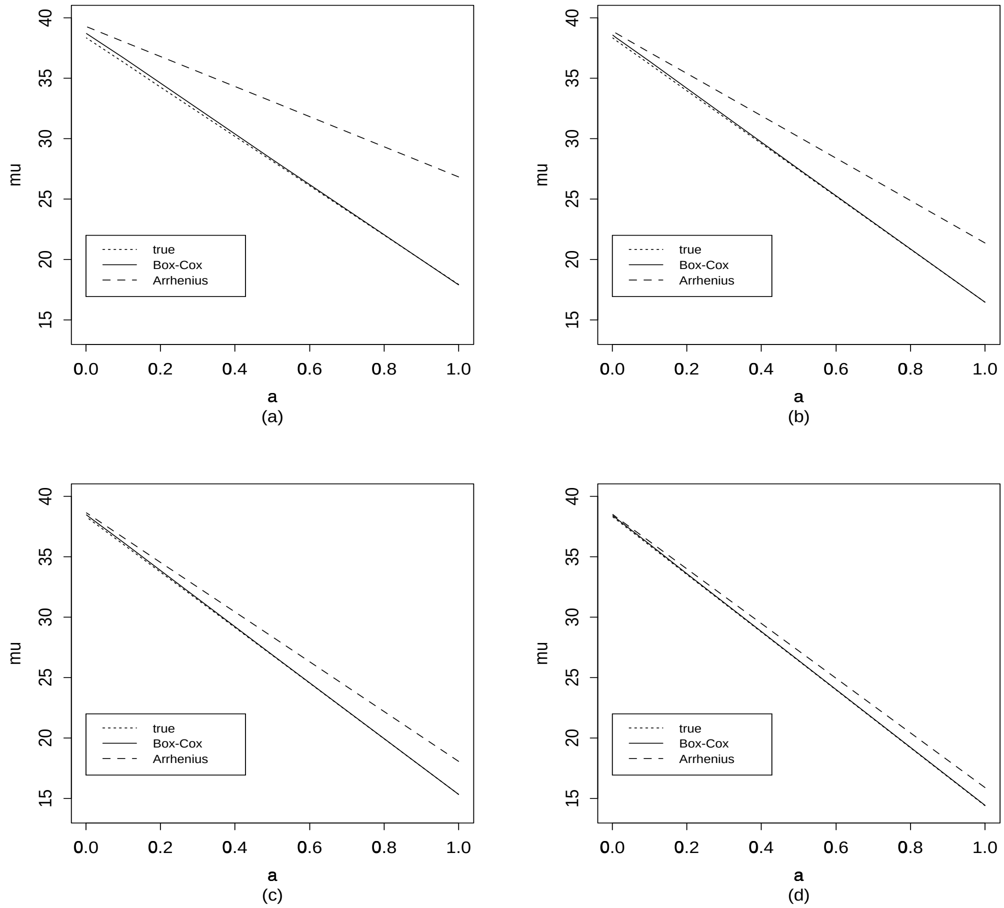

Figure 3.2: Extrapolation results at several T levels when the true model is $\mu = 38.5 + 5/T - 5a[log(T)]$. (a) $T = 60^oC$; (b) $T = 80^oC$; (c) $T = 100^oC$; (d) $T = 120^oC$.

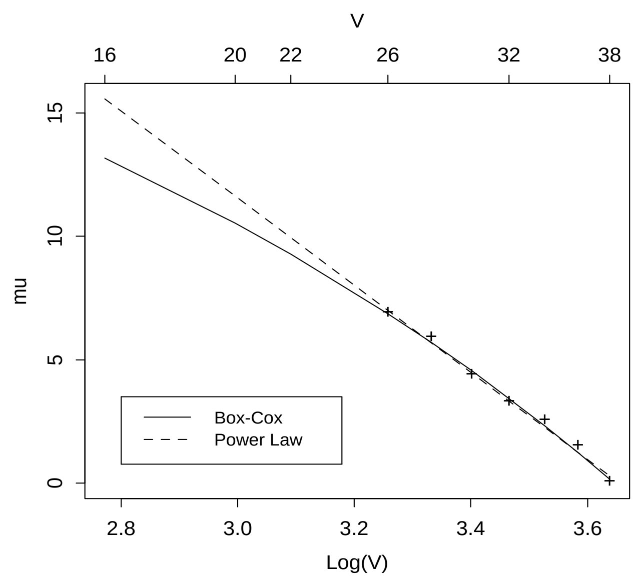

Figure 3.3: The breakdown time data from Nelson (1970). The solid curve and the dashed line represent the fitted models by the Box-Cox transformation method and the power law model, respectively. "+" points denote $\{(log(V_i), \hat{\mu}_i)\}$.