

Intensity Based Nonparametric Image Registration

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ABSTRACT

Image registration is used widely in applications for mapping one image to another. Existing image registration methods are either feature-based or intensity-based. Feature-based methods first extract relevant image features, and then find a geometrical transformation that best matches the two corresponding sets of features extracted from the two images. Because identification and extraction of image features is often a challenging and time-consuming process, intensity-based image registration, by which the mapping transformation is estimated directly from the observed image intensities of the two images, has received much attention recently. In the literature, most existing intensity-based image registration methods require a parametric form of the mapping transformation, which is restrictive for certain applications. In this paper, we propose an intensity-based image registration method without requiring such a parametric form. By this method, the mapping transformation can be nonparametric, and it can even be discontinuous at certain places in the design space. Numerical examples show that it is effective in various applications.

Categories and Subject Descriptors

I.4 [Computing Methodologies]: Image Processing and Computer Vision; I.3 [Computing Methodologies]: Computer Graphics

General Terms

Design

Keywords

Degeneration, edge detection, local smoothing, nonparametric transformation

1. INTRODUCTION

Image registration aims to geometrically match up images or image volumes for structure localization, difference

detection, and other purposes. It is widely used in medical imaging, remote sensing, finger print or face recognition, and so forth. In medical imaging, for instance, a common application of this technique is to integrate useful information from different sources (e.g., CT, PET, SPECT, X-ray, ultrasound, and MRI images [7], [9]), or to register images obtained at different times [13]. It has become an important tool for improving the quality of certain image-based technologies [8].

Mathematically, the image registration problem is often described as follows. Let $Z_R(x, y)$ and $Z_M(x, y)$ be a reference image and an image to register with, respectively. Because $Z_M(x, y)$ is usually a geometrically altered version of $Z_R(x, y)$, in the literature, $Z_M(x, y)$ is often called a moved image. Then, the major goal of image registration is to find a geometrical transformation $\mathbf{T}(x, y) = (T_1(x, y), T_2(x, y))$ such that $Z_M(\mathbf{T}(x, y))$ is as close to $Z_R(x, y)$ as possible. Thus, the image registration problem can be formulated as the following maximization problem:

$$\mathbf{T}_{opt} = \arg \max_{\mathbf{T} \in \mathcal{T}} S(Z_R, Z_M(\mathbf{T})), \quad (1)$$

where $\mathbf{T}_{opt}$ denotes the optimal transformation for matching $Z_R(x, y)$ and $Z_M(x, y)$, S is a selected metric for measuring similarity between two images, and $\mathcal{T}$ is a space of all possible transformations considered. See [12] for a discussion about image registration in MRI applications, and for a description about various existing similarity metrics.

In the literature, there are two types of image registration methods. The first type first selects N corresponding *feature* points in the two images, respectively, and then finds a geometrical transformation to best match the two sets of feature points [3]. To this end, landmarks or control points are often the preferred features and they can be selected manually or automatically by a computer [16]. Other commonly used features include edge lines or curves, which are often detected by gradient-based methods, and regions, centroids or templates, which are usually determined by ways of thresholding and segmentation [14]. In certain applications, however, feature selection is a time-consuming and challenging process with much arbitrariness involved. In such cases, rather than selecting features for image matching, we would prefer to search for a geometrical transformation such that Z_R and Z_M best match each other, in terms of a similarity metric defined directly by the observed image intensities, as described in (1). This type of *intensity-based* image registration (IBIR) procedures is flexible to use, and has become popular in various applications.

Most existing IBIR procedures assume that the transfor-

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mation $\mathbf{T}$ follows a parametric model. A widely used 2-D parametric transformation is defined by

$$\begin{cases} T_1(x, y) &= \alpha(x \cos \Delta\phi + y \sin \Delta\phi) + \Delta x \\ T_2(x, y) &= \alpha(-x \sin \Delta\phi + y \cos \Delta\phi) + \Delta y, \end{cases} \quad (2)$$

where $(\Delta x, \Delta y, \Delta\phi)$ are three motion parameters and α is a scaling parameter. Model (2) describes rigid-body motions (i.e., distance between any two points on an object is unchanged during the motion) when $\alpha = 1$. It has been commonly used in applications [4] for various reasons, including the ease of implementation and computation, and its feature preservation property (e.g., a line maps to a line). Recently, much research progress has been made to generalize model (2). For instance, free-form deformation (DDF) techniques are getting popular in the image registration community, due to their relatively minor computational complexity and other salient properties [15]. DDF methods usually approximate the transformation $\mathbf{T}$ using B-splines with knots chosen properly beforehand. Diffeomorphic image registration also attracts much attention recently, by which the transformation $\mathbf{T}$ is determined by solving a differential equation [2] [1]. Klein et al. [6] have made a comprehensive evaluation of 14 commonly used IBIR algorithms using various test images.

Methods based on parametric models are usually efficient when the parametric models are valid. But, validity of such models should be justified properly in applications, and the proper model justification itself would be challenging and is currently lacking. In this paper, we try to tackle the IBIR problem without imposing any parametric form on $\mathbf{T}$. A nonparametric IBIR procedure is proposed, which matches two images by local smoothing and by using certain image features implicitly. Our proposed procedure is described in detail in Section 2. Its numerical performance is evaluated in Section 3. Certain remarks conclude the article in Section 4.

2. PROPOSED METHOD

Description (1) of the image registration problem is commonly used in the medical imaging and computer sciences communities. A statistically more precise description of the problem might be as follows. Let R and M be two images to register. Their true image intensity functions are denoted as $R(x, y)$ and $M(x, y)$. It is assumed that they have the following relationship:

$$M(T_1(x, y), T_2(x, y)) = R(x, y), \text{ for } (x, y) \in \Omega, \quad (3)$$

where Ω is the design space of the two images and $\mathbf{T}(x, y) = (T_1(x, y), T_2(x, y))$ is a geometrical transformation. IBIR tries to estimate $\mathbf{T}(x, y)$ from observed image intensities of the two images defined below.

$$\begin{aligned} Z_M(x_i, y_j) &= M(x_i, y_j) + \varepsilon_M(x_i, y_j), \\ Z_R(x_i, y_j) &= R(x_i, y_j) + \varepsilon_R(x_i, y_j), \\ &\text{for } i, j = 1, 2, \dots, n, \end{aligned} \quad (4)$$

where $\{(x_i, y_j)\}$ are pixel locations in Ω , and $\{\varepsilon_M(x_i, y_j)\}$ and $\{\varepsilon_R(x_i, y_j)\}$ are i.i.d. random errors in the two images with mean 0 and variances σ_M^2 and σ_R^2 , respectively. Nonparametric intensity-based image registration (NIBIR) tries to estimate $\mathbf{T}(x, y)$ from the observed image intensities, without imposing any parametric form on $\mathbf{T}(x, y)$. In (4), we assume that the two observed images contain pointwise

noise only, for simplicity. In practice, they may also contain spatial blur and other degradations [11]. NIBIR in latter cases could be much more challenging and is not discussed here.

At a given point $(x, y) \in \Omega$, we can write

$$\begin{pmatrix} T_1(x, y) \\ T_2(x, y) \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} b(x, y) \\ c(x, y) \end{pmatrix},$$

where $b(x, y) = T_1(x, y) - x$ and $c(x, y) = T_2(x, y) - y$. Then, estimation of $\mathbf{T}(x, y)$ is equivalent to estimation of $(b(x, y), c(x, y))$. If both $b(x, y)$ and $c(x, y)$ are small (i.e., $\|\mathbf{T}(x, y) - (x, y)\|$ is small) and M has first-order partial derivatives at (x, y) , then by the Taylor's expansion, we have

$$M(T_1(x, y), T_2(x, y)) = M(x, y) + M'_x(x, y)b(x, y) + M'_y(x, y)c(x, y) + o(\|\mathbf{T}(x, y) - (x, y)\|), \quad (5)$$

where $\|\cdot\|$ denotes the Euclidean norm. By (3) and (5), $R(x, y)$ can be approximated well by $M(x, y) + M'_x(x, y)b(x, y) + M'_y(x, y)c(x, y)$. Therefore, $(b(x, y), c(x, y))$ can be chosen such that the approximation residual

$$R(x, y) - [M(x, y) + M'_x(x, y)b(x, y) + M'_y(x, y)c(x, y)]$$

is small. In reality, however, both $R(x, y)$ and $M(x, y)$ are unobservable. What we observed are image intensities $\{Z_M(x_i, y_j)\}$ and $\{Z_R(x_i, y_j)\}$ defined in (4), which have random noise involved. To smooth out random noise while estimate $(b(x, y), c(x, y))$, we adopt the idea of local linear kernel (LLK) estimation [5] as follows. Consider a circular neighborhood of (x, y) with radius h_n , denoted as $O(x, y; h_n)$. Then, if $M'_x(x, y)$ and $M'_y(x, y)$ are replaced by their LLK estimators, denoted as $\widehat{M}'_x(x, y)$ and $\widehat{M}'_y(x, y)$, $b(x, y)$ and $c(x, y)$ can be estimated by the solution of the following minimization problem:

$$\min_{b(x, y), c(x, y)} \sum_{i, j=1}^n \left[Z_M(x_i, y_j) - Z_R(x_i, y_j) + \widehat{M}'_x(x_i, y_j)b(x, y) + \widehat{M}'_y(x_i, y_j)c(x, y) \right]^2 K_{h_n}, \quad (6)$$

where $K_{h_n} = K(\frac{x_i - x}{h_n}, \frac{y_j - y}{h_n})$, and K is a bivariate density kernel function with unit circular support. The minimization problem (6) searches for estimators of $b(x, y)$ and $c(x, y)$ such that the weighted sum of approximation residual squares reaches the minimum, and the weights are determined by the kernel function K . Usually, $K(u, v)$ is chosen to be a decreasing function of $\sqrt{u^2 + v^2}$. Therefore, if a pixel $(x_i, y_j) \in O(x, y; h_n)$ is farther away from the given point (x, y) , then the corresponding approximation residual at (x_i, y_j) would receive less weight, which is intuitively reasonable, because observed image intensities at (x_i, y_j) generally provide less information about $(b(x, y), c(x, y))$, compared to observed image intensities at pixels closer to (x, y) . It is not difficult to check that problem (6) has the following solution

$$\begin{pmatrix} \widehat{b}(x, y) \\ \widehat{c}(x, y) \end{pmatrix} = \frac{\begin{pmatrix} K_{22} & -K_{12} \\ -K_{12} & K_{11} \end{pmatrix} \begin{pmatrix} K_{11}^* \\ K_{22}^* \end{pmatrix}}{K_{11}K_{22} - K_{12}^2}, \quad (7)$$

where

$$\begin{aligned}
K_{11} &= \sum_{i,j=1}^n [\widehat{M}'_x(x_i, y_j)]^2 K_{h_n}, \\
K_{22} &= \sum_{i,j=1}^n [\widehat{M}'_y(x_i, y_j)]^2 K_{h_n}, \\
K_{12} &= \sum_{i,j=1}^n \widehat{M}'_x(x_i, y_j) \widehat{M}'_y(x_i, y_j) K_{h_n}, \\
K_1^* &= \sum_{i,j=1}^n [Z_R(x_i, y_j) - Z_M(x_i, y_j)] \widehat{M}'_x(x_i, y_j) K_{h_n}, \\
K_2^* &= \sum_{i,j=1}^n [Z_R(x_i, y_j) - Z_M(x_i, y_j)] \widehat{M}'_y(x_i, y_j) K_{h_n}.
\end{aligned}$$

From the above description, the transformation $\mathbf{T}(x, y)$ can be well estimated by

$$\widehat{\mathbf{T}}(x, y) = (x, y) + (\widehat{b}(x, y), \widehat{c}(x, y)) \quad (8)$$

with $(\widehat{b}(x, y), \widehat{c}(x, y))$ defined by (7) in cases when

- (i) $\|\mathbf{T}(x, y) - (x, y)\|$ is small such that the first-order approximation to $M(T_1(x, y), T_2(x, y))$ in (5) is valid,
- (ii) M has first-order partial derivatives at (x, y) , and
- (iii) the denominator in the right-hand-side of (7) is not too small.

The above conditions (i) and (ii) imply that the estimator defined by (7) and (8) may not estimate $\mathbf{T}(x, y)$ well at places where the transformation is relatively large or where M is not smooth (e.g., edge locations of M). Condition (iii) implies that the estimator may not be well defined at places where the following equation holds:

$$[M'_x(x, y)]^2 [M'_y(x, y)]^2 - [M'_x(x, y) M'_y(x, y)]^2 = 0, \quad (9)$$

because $\frac{1}{\pi n^2 h_n^2} K_{11}$, $\frac{1}{\pi n^2 h_n^2} K_{22}$, and $\frac{1}{\pi n^2 h_n^2} K_{12}$ can be regarded as local constant kernel estimators of $[M'_x(x, y)]^2$, $[M'_y(x, y)]^2$, and $M'_x(x, y) M'_y(x, y)$, respectively, which are all statistically consistent under some regularity conditions.

Mathematically, it can be proved that M satisfies equation (9) in a neighborhood $O(x, y)$ of a given point (x, y) if and only if there is a continuously differentiable function ψ and a constant ρ such that

$$M(x', y') = \psi(\rho x' + y'), \text{ for any } (x', y') \in O(x, y). \quad (10)$$

Intuitively, if M satisfies (10) in $O(x, y)$, then its intensity levels are the same on the line segment $\rho x' + y' = \rho_0$ in $O(x, y)$, for any appropriate constant ρ_0 . In such cases, the bivariate function M is degenerate locally in $O(x, y)$, and it is impossible to uniquely determine $\mathbf{T}(x, y)$ because any small move along the line direction would not change the value of $M(x', y')$ for any $(x', y') \in O(x, y)$. In this paper, a point $(x, y) \in \Omega$ is called a *local degenerate point* of M if M has partial derivatives at (x, y) and there exists a neighborhood $O(x, y)$ such that equation (9) holds. Other points are called *local non-degenerate points*. Therefore, around local degenerate points, the image registration problem is actually not well defined.

Based on the above analysis, we propose a NIBIR procedure consisting of the following several steps:

Step 1 Detect edge pixels for the observed moved image Z_M using an edge detector. See Chapter 6 in [10] for a discussion about existing edge detectors.

Step 2 At a given point (x, y) , consider its circular neighborhood with radius r_1 , denoted as $O(x, y; r_1)$. If the detected edge pixels in $O(x, y; r_1)$ is smaller than nr_1 , then (x, y) is regarded as a continuity point of M . In such cases, if the denominator of (7) is larger than or equal to a threshold value u_n , then (x, y) is regarded as a local non-degenerate point and $\widehat{\mathbf{T}}(x, y)$ is defined by (7) and (8).

Step 3 If the detected edge pixels in $O(x, y; r_1)$ is larger than or equal to nr_1 , then there might be an edge segment in $O(x, y; r_1)$. In such cases, $\widehat{\mathbf{T}}(x, y)$ is searched as follows. For any pixel (x', y') in $O(x, y; r_1)$ of the moved image Z_M , consider its circular neighborhood $O(x', y'; r_2)$. Compute the mean squared difference (MSD) as follows.

$$MSD((x', y'); (x, y)) = \frac{1}{\widetilde{N}} \sum_{(x'+s, y'+t) \in O(x', y'; r_2)} [Z_M(x' + s, y' + t) - Z_R(x + s, y + t)]^2,$$

where $\widetilde{N}$ is the number of pixels in $O(x', y'; r_2)$. Then, $\widehat{\mathbf{T}}(x, y)$ is defined to be the minimizer of

$$\min_{(x', y') \in O(x, y; r_1)} MSD((x', y'); (x, y)).$$

See Figure 1 for a demonstration.

Step 4 In $O(x, y; r_1)$, if the detected edge pixels is smaller than nr_1 and the denominator of (7) is smaller than the threshold u_n , then (x, y) is regarded as a local degenerate point of M . In such cases, $\widehat{\mathbf{T}}(x, y)$ is defined as follows. First, we find a closest local non-degenerate point $(x^{(1)}, y^{(1)})$ of (x, y) in Z_M . Second, we find a closest detected edge pixel $(x^{(2)}, y^{(2)})$ to (x, y) in Z_M . Third, compute $MSD(\widehat{\mathbf{T}}(x^{(1)}, y^{(1)}); (x, y))$ and $MSD(\widehat{\mathbf{T}}(x^{(2)}, y^{(2)}); (x, y))$. Finally, define $\widehat{\mathbf{T}}(x, y)$ to be $\widehat{\mathbf{T}}(x^{(1)}, y^{(1)})$ if

$$MSD(\widehat{\mathbf{T}}(x^{(1)}, y^{(1)}); (x, y)) \leq MSD(\widehat{\mathbf{T}}(x^{(2)}, y^{(2)}); (x, y)),$$

and define $\widehat{\mathbf{T}}(x, y)$ to be $\widehat{\mathbf{T}}(x^{(2)}, y^{(2)})$ otherwise.

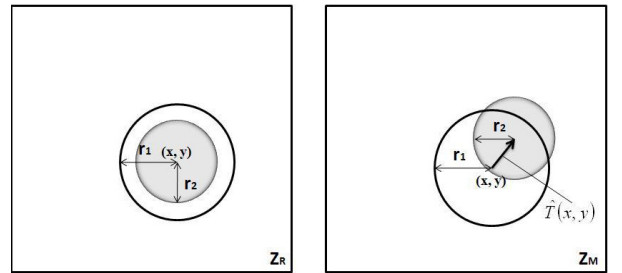


Figure 1: A demonstration of defining $\widehat{\mathbf{T}}(x, y)$ around detected edge pixels.

3. NUMERICAL STUDY

In this section, we present a numerical example to evaluate the performance of the proposed NIBIR procedure. The evaluation is made in comparison with the following two state-of-the-art IBIR methods: the directly manipulated free-form deformation (DMFFD) method [15] and the symmetric diffeomorphic image normalization (SyN) method [1].

As suggested in the related papers, three similarity metrics, including the mean squared difference (MSD), cross-correlation (CC), and mutual information (MI), are used respectively in the DMFFD method, and two similarity metrics, including the CC and pure cross correlation (PCC), are used respectively in the SyN method. These existing methods are implemented using the software package ANTS available at <http://www.picsl.upenn.edu/ANTS/>.

A reference image and a moved image are shown in Figure 2. The reference image mainly contains a railway and a ball; the ball moved its position in the moved image. Therefore, in this example, the geometrical transformation $\mathbf{T}$ takes non-zero values mainly in the region of the reference image that contains the ball. It is thus a discontinuous transformation.

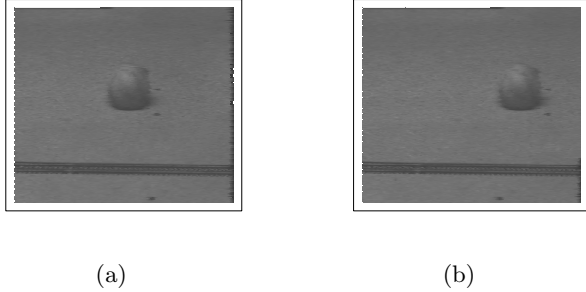


Figure 2: (a) A reference image containing a railway and a ball. (b) A moved image.

Then, the proposed NIBIR procedure and the existing methods DMFFD-MSD, DMFFD-CC, DMFFD-MI, SyN-CC, and SyN-PCC are respectively applied to this example. In NIBIR, the kernel function $K(u, v)$ is chosen to be the truncated bivariate Gaussian density function with support $\{(u, v) : u^2 + v^2 \leq 1\}$. For a given method, we search for an estimator of the geometrical transformation, denoted as $\hat{\mathbf{T}}(x, y) = (\hat{T}_1(x, y), \hat{T}_2(x, y))$, by trying all possible values of its procedure parameters such that the root residual mean square (RRMS) value reaches the minimum, where

$$\text{RRMS} = \left\{ \frac{1}{n^2} \sum_{i,j=1}^n \left[Z_R(x_i, y_j) - Z_M(\hat{\mathbf{T}}(x_i, y_j)) \right]^2 \right\}^{1/2}.$$

The restored reference image of the given method is then defined by $Z_M(\hat{\mathbf{T}}(x, y))$. Figure 3(a)-(f) show the restored reference images of the six related methods. From the plots, it can be seen that the proposed NIBIR procedure restores the original position of the ball well, and all five competing methods can not restore the reference image properly. Our explanation of these results is that all five competing methods can only handle the cases when the geometrical transformation $\mathbf{T}(x, y)$ is continuous in the entire design space, while our proposed NIBIR procedure defines $\hat{\mathbf{T}}(x, y)$ locally and has the flexibility to allow $\mathbf{T}(x, y)$ to be discontinuous. To further see the difference among the results of the six methods, we present the residual images of the six methods in Figure 3(g)-(l). For a given method, the residual image is defined to be $Z_R(x, y) - Z_M(\hat{\mathbf{T}}(x, y))$. From the plots, it can be seen that the residual image of the NIBIR procedure is almost empty, while the residual images of the other five methods all show the ball clearly. The minimal RRMS val-

ues of the six methods are 10.929, 12.852, 13.402, 14.416, 14.053, and 15.121, respectively.

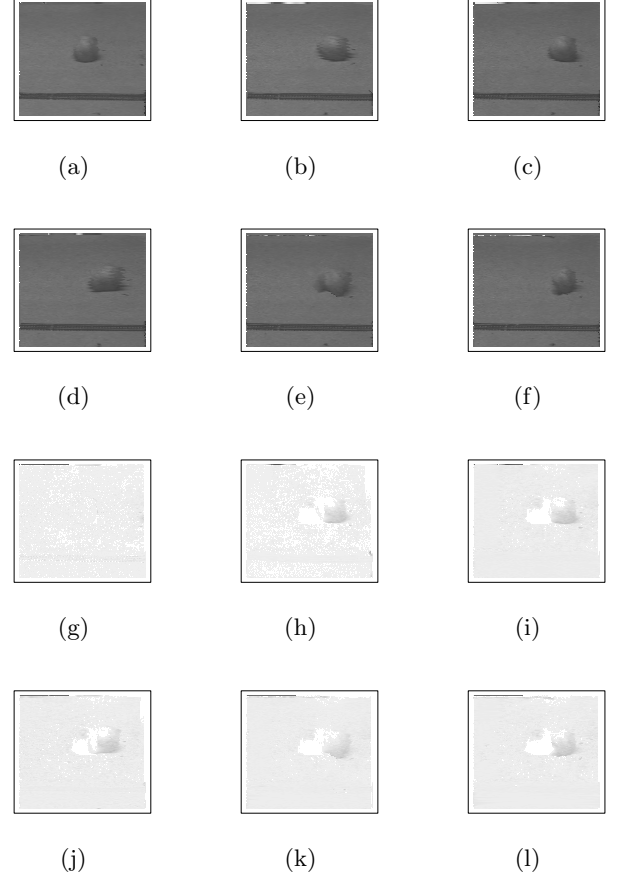


Figure 3: (a)-(f) Restored reference images of NIBIR, DMFFD-MSD, DMFFD-CC, DMFFD-MI, SyN-CC, and SyN-PCC, respectively. (g)-(l) Residual images of the six methods.

4. CONCLUSIONS

In this paper, we propose an intensity-based image registration method which allows the geometrical transformation $\mathbf{T}(x, y) = (T_1(x, y), T_2(x, y))$ to be nonparametric. From the numerical example shown in Section 3, we can see that it also allows $\mathbf{T}(x, y)$ to be discontinuous in the design space. In the proposed method, there are still many issues that need to be addressed in our future research. For instance, in the proposed method, there are several parameters, including h_n , u_n , r_1 , r_2 and other parameters used in edge detection. Appropriate selection of these parameters should be important for the entire procedure, which has not been properly addressed yet. By the nature of the image registration problem, if the reference image Z_R and the moved image Z_M are switched, then the resulting geometrical transformation should be exactly the inverse of $\mathbf{T}(x, y)$. However, by the proposed method, the resulting estimators of the two geometrical transformations may not have this inverse relationship, which should be further studied in our future research.

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