Problem-Solving and Reasoning  
November 8, 2007

Thinking  
• Involves “consciousness” of thought, typically of products, not necessarily processes  
• Thinking involves “directedness” of thought  
• Convergent vs. divergent thinking  
• Knowledge-lean problems vs. expert problems

The Problem with problem-solving research  
• “In field research, there is often too much [complexity] to allow for definitive conclusions, and in laboratory research, there is usually too little complexity to allow for any interesting conclusions”  
  Brehmer & Dörner (1993)  
  Computers in Human Behavior, 9, 171-183

Salient differences between puzzle problems and real-world problems  
1 Puzzles  
   • unfamiliar  
   • involve no prior knowledge  
   • all necessary info. is present in the problem statement  
   • requirements are unambiguous

2 Real-world problems  
   • familiar  
   • require prior knowledge  
   • necessary information often absent  
   • Solver must ask ‘what is the goal?’

Applications Examples – Production Systems  
• Procedures to assess drug safety  
• Inventory control methods for industry  
• Modeling energy use and environmental impact  
• Defense strategy scenarios  
• Expert systems for medical diagnosis

Problem Examples  
• Water jug problem  
• Nine-dot problem  
• Candle-wall problem  
• Missionaries and cannibals  
• Tower of Hanoi  
• Two-string problem
**Gestalt Viewpoint**
- Problem-solving is both reproductive and productive
- Reproductive PS involves re-use of previous experience (can be beneficial or detrimental)
- Productive problem-solving is characterized by restructuring and insight
- Insight accompanied by subjective “ah-ha”

**Gestalt Contributions**
- Problem-solving: trial-and-error or otherwise?
- Perception more than just association
- Functional Fixedness can hinder problem-solving (candle problem) Problem restructuring:
  - productive
- Development of insight
- Implication: importance of problem representation

**Ohlsson’s Insight Theory**
- Problem-space reinterpretation of gestalt findings
  - multiple mental representations of the same problem
  - specific knowledge operators needed are retrieved from memory
  - current representation of the problem acts as a memory probe
  - impasses in problem-solving are solved through ‘re-representation’
    - elaboration
    - constraint relaxation
    - restructuring or recategorization

**Routine v. Insight Problems**
- Key Concept: insight and trial-and-error (routine) problems involve subjectively different experiences
- Key Debate: “Special Process” vs. “Business as Usual”
- Routine: problem-solvers good at predicting their success; monitor accurately how close they are to solution
- Insight: problem-solvers poor at predicting success; can’t monitor closeness to solution
  - “What can move large logs but cannot move a small nail?”

**Neurobiology of Insight**

**Information-Processing Approach to Problem-Solving**
- Problem-Space Theory
  - solving a problem involves negotiating alternative paths to a solution
  - initial state is linked to goal state by a path
  - knowledge states are produced by the application of mental operators
  - algorithms vs. heuristics are used to move along the path
- limited processing resources provide constraints on the degree to which multiple moves can be considered

20 Important Ideas in Problem-Space Theory
• **Problem-space** refers to the abstract structure of a problem
• **Operators** are specific knowledge structures that transform data
• **Algorithm**: method or procedure
• **Heuristics**: strategies, “rules of thumb”
  - **means-end analysis**: calculate difference between current state and goal; create a subgoal to reduce that difference; select an operator that will solve this subgoal
• **Subgoal structure** - essentially are short- and long-term goals

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25 Analysis of Water-Jug Problem
• Subjects look ahead only 1 move
• Moves evaluated using means-end analysis
• Anti-looping heuristic
• Limitations to working memory
• May use LTM

26 Extensions of PST
• Ill-defined problems
• Novice vs. expert problem-solvers
  - impact of knowledge structures on problem-solving

27 Progress Monitoring Theory
• MacGregor, Ormerod, and Chronicle (2001)
• Two general problem-solving heuristics
  - Maximisation heuristic
  - Progress monitoring
• Criterion failure can be important in obtaining a solution

28 Four versions of the eight-coin problem. The dark shading indicates that one coin was on top of another coin. The figures on the right have valid two-dimensional moves, whereas those on the left do not. From Ormerod et al. (2002). Copyright © 2002 by the American Psychological Association. Reprinted with permission.

29 Evaluation
• Insight appears to be dependent on
  - Constraint relaxation
  - Combined with criterion failure
- Problem solvers who realise that means–ends analysis is proving unsuccessful are more responsive to changing their strategy than are those for whom means–ends analysis is at least partially successful
- Previous experience with related problems is also important

30 Problem Isomorphs
- Similar formal structure of two problems
- Reasoning by analogy
- Similarities often very difficult to detect if the problems do not have identical structure (an impediment to generalization)
- Military vs. radiation problem

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33 Mental Models in Problem-Solving
- Developing an understanding of the formal structure of the problem
- Imagistic and propositional processing
- Testing out formal implications of problem by seeking examples and counterexamples

34 Flagpole Problem
- Two flagpoles are standing, each 100 feet tall. A 150-foot rope is strung from the top of one of the flagpoles to the top of the other and hangs freely between them. The lowest point of the rope is 25 feet above the ground. How far apart are the flagpoles?

35 The Singles Bar
- While sitting in a club where all single men tell the truth and all married men lie, a woman is approached by three men. She asks the first guy if he is married, but the music is so loud that she can't hear his answer. So she turns to the second guy, who tells her, "The first guy said, 'I am married,' but he really is single." Then she turns to the third guy, who says, "The second guy is single." Determine the marital status of each of the three men.

36 Obstacles to Problem-Solving
- Mental sets, entrenchment, and fixation (viewing the problem from the "dominant paradigm")
- Negative transfer
- Memory load/interference – importance of incubation

37 Types of Expertise
- Routine ("competence")
  - ability to solve familiar problems
  - relies on schemata
- Adaptive ("capacity")
  - development of ad hoc procedures to solve unfamiliar problems
  - ability to evaluate seems important
Chess Expertise

- Chess experts remember previous experiences with board position, and use this to plan moves
  - can remember arranged positions from real games better than novices, but are equal to novices in memory for random arrangements
- Experts constructed larger memory “chunks” than novices
- Experts have knowledge about “strength” of position

Decision-Making and Reasoning

- 85% of people rate “h” as more likely than “f”
- Fallacy in reasoning: probability of “h” cannot strictly be higher than “f”, since “h” is a subset of “f”

Reasoning Research

- Goal of judgment and decision-making is to select from choices
- Goal of reasoning is to draw conclusions deductively from principles (e.g., applying laws of physics to determine power of an engine) and inductively from evidence (e.g., using safety statistics to draw inferences about the safety of a particular car)

Decision-Making

Classical Decision Theory

- Assumes “rational man” - based on economics
  - fully informed regarding options and outcomes
  - sensitive to subtle distinctions between options
  - fully rational with regard to choice of options

Expected Utility Theory

- Seek to maximize positive utility (pleasure)
- Seek to minimize negative utility (pain)
- Components:
  - subjective utility: based on individual’s judged weightings of utility
  - subjective probability: based on individual’s judged weightings of probability

Which job should I take?

Company A: 50% chance of a 20% salary increase the first year
Company B: 90% chance of getting a 10% salary increase the first year

1 Classical Decision Theory

- Calculate expected value for each option:
  - Company A: 5 x 0.2 = 10
  - Company B: 0.9 x 1 = 0.9
- Perform similar calculations for other factors (e.g., health insurance, severance package, vacation allowance, job satisfaction)
- Assuming other things equal, choose job with Company A

2 Expected Utility Theory
• Assign individual subjective weighting to various factors (salary, health insurance, etc.)
• Assign individual subjective weights to various probabilities of obtaining positive utility (strategy important)
• Calculate: $\sum [p(\text{pos})] - \sum [p(\text{neg})]
• Choose Company based on the sum of expected positives - negatives

48  Prospect Theory
(Kahneman & Tversky)
• Describes how individuals evaluation losses and gains
• Two stages
  – Stage I: 
  – Stage II: Evaluation
• Explains a variety of economic behaviors
  – Status quo effect – insurance example
  – Endowment effect – coffee cup example
  – Sunk cost effect – vacation example

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50 Prospect Theory – Example Applications
• Understanding white collar crime: covering up minor crimes (failure to cut losses)
• Iraq war, other examples of organizational inertia – sunk cost effect?
• Stock investing - Why do so many financial investors hold onto a stock that has plummeted far more frequently than they keep a stock that has risen sharply, or that has maintained a steady price?
• Health decision-making: risk-taking in bad situations (e.g. HIV/AIDS)

51 Example (Prospect Theory)
• 600 people are at risk of dying of a particular disease. Vaccine A could save 200 of these lives. For Vaccine B, there is a .33 likelihood that all 600 people would be saved, but a .66 likelihood that all 600 people will die. Would you choose A or B? (most choose A)
• 600 people are at risk of dying of a particular disease. If Vaccine C is used, 400 of these people will die. If Vaccine D is used, there is a .33 likelihood that no one will die, but a .66 likelihood that all 600 people will die. Would you choose C or D? (most choose D)

52 Satisficing (Simon)
• Reaching “acceptable” goals
• Notion of “bounded rationality”: rationality, but within limits
• Do not consider total range of options, but consider options one by one until one meets our minimum standards of acceptability
• Probably don’t reach optimal solution, but also don’t spend eternity searching for one

53 Elimination by Aspects (Tversky)
• Consider one aspect (attribute) of available options
• Form minimum criterion for that aspect
• Eliminate all options that don’t conform to minimum criterion
• Then select a second aspect…and so on

54 Models of Probability Judgments
• Descriptive: how people reach decisions (naturalistic observation)
• Normative: how a decision should be made using unlimited resources.
  – Bayes’ Theorem:

\[
P(E|H) \times P(H) = P(E|H) \times P(H) + P(E|\text{not } H) \times P(\text{not } H)
\]
\[
\begin{align*}
= \frac{(0.9 \times 0.05)/0.95}{(0.9 \times 0.05)+0.95} \\
= 0.32
\end{align*}
\]

55 **Bayes Theorem Applied**
- In previous example, two types of probabilities exist:
  - "prior probability": probability that event will occur given similar prior circumstances (e.g., \( p = 0.05 \) that your friend will invite your ex-husband to the party)
  - "conditional probability": probability that new information is true if a particular hypothesis is true (e.g., \( p = 0.90 \) that the car you see parked belongs to him)

56 **Are we accurate probability calculators?**
- Probably not...we're more conservative
  - Edwards (1968): drawing chips, with replacement, from one of two bags with 70/30 mix of red/white chips. If first chip is red, what's the probability that the second chip will also be red? Actual \( p = 0.70 \) (subjects say \( p = 0.60 \))
  - Meehl’s criticisms of clinical decision-making and the clinical-actuarial debate

57 **Probability Judgments**
- Three candidates, A, B, and C are running for Mayor of Gainesville. In 6 separate polls, A led B five times. In 18 polls, C led B 9 times. In a comparison of A and C, who is more likely to win?
- It is known that 5% of the population is affected by rubadubitis. A new diagnostic test gives true positives of the disease 85% of the time, but has a 10% false positive rate. Bub has tested positive. What is the probability that he has rubadubitis?

58 **Common Heuristics in Probability Judgments**
- **Frequency Heuristic**: making use of number of occurrence, rather than probability of occurrence
  - candidate example: C has more wins, but A has greater proportion of wins (5/6); most people choose C

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60 **Common Heuristics (cont’d)**
- **Representativeness Heuristic**: making choices based on how similar/representative a person or sample is, rather than relying on calculated probability
  - fail to use conjunctive rule: Linda is regarded as “representative” of a feminist, so most people choose “b”
  - fail to use baserates: rubadubitis example, estimates are around .85 (actual answer is .31)

61 **Common Heuristics (cont’d)**
- **Availability Heuristic**: using most salient, or apparent answer to guide judgment
  - Which is more likely: death by tornado or death by asthma? (asthma)
  - Is the letter “k” more likely to occur in the first or third position in English words? (3rd)
- Conclusion: people aren’t very good at calculating probabilities; they rely on heuristics

62 **Heuristics and Biases (Kahneman & Tversky)**
- People commonly use short-cuts (heuristics)
- Heuristics lighten cognitive load, but lead to greater biases and errors
- Example heuristics:
  - REPRESENTATIVENESS: how representative instance is of universe
  - AVAILABILITY: how easily instances are called to mind
Examples
• All families having exactly 6 children in Pleasantville were surveyed. In 72 families, the exact birth order was GBGBBG. What is your estimate of the number of families in which the birth order was BGBBBB?
• Are there more words in English that begin with the letter R, or that have R as their third letter?

Examples (cont’d)
• In a 2000 word passage, estimate the number of words of the form ‘________ing’. Estimate the number of words of the form ‘________n_’.
• What percentage of men in a health survey have had one or more heart attacks? What percentage of surveyed men both are over 55 and have had one or more heart attacks? (conjunction fallacy)

Anchoring and Framing Effects
• Anchoring effect (ans = 40,320)
  - Estimate: 8x7x6x5x4x3x2x1 (2,250)
  - Estimate: 1x2x3x4x5x6x7x8 (512)
• Framing effects
  - the way that options are presented affects option selection
    • risk aversion when presented with a gain options (pick small but certain gain over large but uncertain one)
    • risk seeking when presented with potential losses (choose large, uncertain loss rather than smaller, certain loss)

Other Judgement Phenomena
• Illusory correlation: tend to see associated characteristics as causally related
• Overconfidence: overvaluing one’s own skill or knowledge (prospective probability judgments)
• Hindsight bias: ability to see signs and events leading up to an outcome

Part II: Reasoning
Truth Tables and Logical Operators
• Concept of propositional calculus (assertion that is either true or false)
• Limited number of operators: not, and, or, if…then, if and only if
• Truth tables chart truth value of proposition by laying out state-of-world possibilities
• Use of conditional logic

Forms of Conditional Reasoning, based on “If P then Q”
• Valid Forms
  - Modus Ponens: P, ∴ Q
  - Modus Tollens: not Q, ∴ not P
• Invalid Forms
  - Affirming the Consequent: Q, ∴ P
  - Denying the Antecedent: not P, ∴ not Q
• Additional or alternative antecedents affect the use of inferential forms
Theories of Reasoning

- Abstract-Rule Theories: reasoning proceeds much like logical proofs
- Domain-Specific-Rule Theories: reasoning based on schematic rules specific to the type of problem (Wason's selection task)
- Model Theories: reasoning proceeds using mental models of the world (syllogisms)
- Bias Accounts: reasoning as a product of nonlogical tendencies (Wason's selection task)

Abstract-Rule Theory

- Natural language premises encoded by a comprehension mechanism; this mechanism is normally rational but can be derailed
- Representation of premises is related to elementary, abstract reasoning rules (e.g., modus ponens)
- If these rules do not produce conclusion, then non-logical processes are invoked
- Types of errors
  - comprehension: premise misconstrued
  - heuristic inadequacy: poor strategy
  - processing: attentional, working memory lapses

Abstract-Rule Account of Invalid Inferences

- Premises are re- or mis-interpreted
- Importance of “co-operative principle” (speaker tells hearer exactly what they think the hearer should know); hearer then makes invalid inferences
  - e.g.: the only way Alicia can get wet is if it rains on her

Status of Abstract-Rule Theory

- Can account for rule-based inference problems and for effects of alternative and multiple antecedents
- Comprehension component underspecified
- Applicable only to propositional reasoning situations

Wason's Selection Task: "turn over only those cards that need to be turned over to verify the rule"

(assume all cards follow the same rule)

- Best strategy: turn over E and 7 (a confirming and disconfirming instance)
- Most subjects turn over E and 4 or E only
- Possible bases for subject behavior:
  - verification of the rule (rather than refutation)
  - response matching (selecting cards mentioned in rule)
- Subjects more accurate with realistic materials, more so for relevant or experienced tasks – stored knowledge important

Domain-Specific Knowledge and Reasoning

- Posit types of situation-specific rules that are used to solve reasoning problems (probabilistically based):
  - specific prior experience
  - schemata for different types of situations (e.g., permissions, obligations)
- Rules have specific form that can be applied in all situations corresponding to that schema

Model Theory

- Three processes:
- **comprehension of premises**: semantics and analogy
- **combining/description**: models of simple premises are combined to form integrated model
- **validation**: search for counterexamples or alternative models disconfirming the conclusion

- Models consume processing resources
- Errors arise from inadequate models

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80 Model Theory (cont’d)
- Valid Inferences
  - develop and “flesh out” models based on propositions
  - working models out may take up processing resources
- Invalid Inferences
  - incorrect initial models (e.g., confusing biconditional with conditional)
  - can account for context effects; additionals serve as counterexamples

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81 Bias Theory