

Exponential Model

$$\begin{cases} \beta = 0 \\ \gamma = 1 \end{cases}$$

$$Y = \begin{cases} 0 & \text{cured} \\ 1 & \text{not cured} \end{cases}$$

T : survival time starting from study entry.

Model: $P(T > t | Y=1) = e^{-\lambda t}$

$$P(T > t | Y=0, a) = \frac{S_0(a+t)}{S_0(a)}$$

$P(Y=0) = p$, $P(Y=1) = 1-p$.
age at entry

Data: (t, δ, a, y)

The observation on Y could be missing.

The observation on Y never equals 0.

(1) If $y=1$ (died due to the disease),
the likelihood is δ or $\lambda e^{-\lambda t}$.

$$L = (1-p) \cdot [\lambda]^\delta e^{-\lambda t}$$

(2) If $y=0$, the likelihood is

$$L = \left[p \frac{S_0(a+t)}{S_0(a)} + (1-p) e^{-\lambda t} \right]^{1-\delta} \cdot \left[p \frac{f_0(a+t)}{S_0(a)} + (1-p) \lambda e^{-\lambda t} \right]^\delta$$

Farwell model:

Replace $\frac{S_0(a)}{S_0(a)}$ by 1

Replace $\frac{f_0(a)}{S_0(a)}$ by 0