1 # Problem-Solving and Reasoning

November 6, 2008

2 The Problem with problem-solving research

"In field research, there is often too much [complexity] to allow for definitive conclusions, and in laboratory research, there is usually too little complexity to allow for any interesting conclusions"

Brehmer & Dörner (1993)
Computers in Human Behavior, 9, 171-183

3 ## Salient differences between puzzle problems and real-world problems

1 • **Puzzles**
  – unfamiliar
  – involve no prior knowledge
  – all necessary info. is present in the problem statement
  – requirements are unambiguous

2 • **Real-world problems**
  – familiar
  – require prior knowledge
  – necessary information often absent
  – solver must ask 'what is the goal'?

4 ## Problem Examples

- Water jug problem
- Two-string problem
- Nine-dot problem
- Candle Box problem
- Missionaries and cannibals
- Tower of Hanoi

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7 Candle Box Problem (Duncker, 1945)

8 ## Gestalt Viewpoint

- Problem-solving is both reproductive and productive
- Reproductive PS involves re-use of previous experience (can be beneficial or detrimental)
- Productive problem-solving is characterized by restructuring and insight
- Insight accompanied by subjective "ah-ha" experience

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10 ## Gestalt Contributions

- Perception more than just association – it involves conceptualization
- *Functional Fixedness* can hinder problem-solving (candle box problem)
- *Problem restructuring:* productive
- Development of *insight*

- Implication: importance of problem *representation*

11 ☐ Information-Processing Approach to Problem-Solving

- Problem-Space Theory
  - solving a problem involves negotiating alternative paths to a solution
  - initial state is linked to goal state by a path
  - knowledge states are produced by the application of mental operators
  - algorithms vs. heuristics are used to move along the path
  - limited processing resources provide constraints on the degree to which multiple moves can be considered

12 ☐ Ohlsson's Insight Theory

- Gestalt findings can be reinterpreted within PST
  - multiple mental representations of the same problem
  - specific knowledge operators needed are retrieved from memory
  - current representation of the problem acts as a memory probe
  - impasses in problem-solving are solved through 're-representation'
    - elaboration
    - constraint relaxation
    - restructuring or recategorization

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15 ☐ Routine v. Insight Problems: A useful distinction?

- Key Concept: insight and trial-and-error (routine) problems involve subjectively different experiences
- Key Debate: "Special Process" vs. "Business as Usual"
- Routine: problem-solvers good at predicting their success; monitor accurately how close they are to solution
- Insight: problem-solvers poor at predicting success; can't monitor closeness to solution
  - "What can move large logs but cannot move a small nail?"

16 ☐ Neurobiology of Insight

17 ☐ Problem Isomorphs

- Similar formal structure of two problems
- Reasoning by analogy
- Similarities often very difficult to detect if the problems do not have identical structure (an impediment to generalization)
- Military vs. radiation problem

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20 ☐ Information-Processing Approach to Problem-Solving

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21 Important Ideas in Problem-Space Theory

- **Problem-space** refers to the abstract structure of a problem
- **Operators** are specific knowledge structures that transform data
- **Algorithm**: method or procedure
- **Heuristics**: strategies, "rules of thumb"
  - **means-end analysis**: calculate difference between current state and goal; create a subgoal to reduce that difference; select an operator that will solve this subgoal
  - **Anti-looping heuristic**: don't go further from the goal than you currently are
- **Subgoal structure** – essentially are short- and long-term goals (interim v. final destinations)

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26 Progress Monitoring Theory (MacGregor, Ormerod, and Chronicle, 2001)

- Two general problem-solving heuristics
  - Maximisation heuristic – making the most headway possible
  - Progress monitoring – assessing rate of progress toward goal
- Criterion failure ("wake up call") causes problem solvers to seek an alternative strategy and can be important in obtaining a solution

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Four versions of the eight-coin problem. The dark shading indicates that one coin was on top of another coin. The figures on the right have valid two-dimensional moves, whereas those on the left do not. From Ormerod et al., (2002). Copyright © 2002 by the American Psychological Association. Reprinted with permission.

28 Evaluation

- Insight appears to be dependent on
  - Constraint relaxation
  - Combined with criterion failure
  - Problem solvers who realize that means–ends analysis is proving unsuccessful are more responsive to changing their strategy than are those for whom means–ends analysis is at least partially successful
- Jones (2003)
  - Previous experience with related problems is also important

29 Mental Models in Problem-Solving

- Developing an understanding of the formal structure of the problem
- Imagistic and propositional processing
- Fleshing out formal implications of problem by seeking examples and counterexamples, or by playing out real-world implications

30 Flagpole Problem

- Two flagpoles are standing, each 100 feet tall. A 150-foot rope is strung from the top of one of the flagpoles to the top of the other and hangs freely between

them. The lowest point of the rope is 25 feet above the ground. How far apart are the flagpoles?

31 ☐ The Singles Bar

- While sitting in a club where all single men tell the truth and all married men lie, a woman is approached by three men. She asks the first guy if he is married, but the music is so loud that she can't hear his answer. So she turns to the second guy, who tells her, "The first guy said, 'I am married,' but he really is single." Then she turns to the third guy, who says, "The second guy is single." Determine the marital status of each of the three men.

32 ☐ Obstacles to Problem-Solving

- Mental sets, entrenchment, and fixation (viewing the problem from the "dominant paradigm)
- Negative transfer
- Memory load/interference – importance of incubation

33 ☐ Decision-Making and Reasoning

34 ☐

35 ☐ Results

- "h" rated as more likely than "f" 85% of the time!
- But Joe, it can't be so! The probability of "h" cannot be higher than "f", since "h" is a subset of "f"

36 ☐ Reasoning Research

- Goal of judgment and decision-making is to select from choices
- Goal of reasoning is to draw conclusions deductively from principles (e.g., applying laws of physics to determine power of an engine) and inductively from evidence (e.g., using safety statistics to draw inferences about the safety of a particular car)

37 ☐ Decision-Making

38 ☐ Classical Decision Theory

- Assumes "rational man" - based on economics
  - fully informed regarding options and outcomes
  - sensitive to subtle distinctions between options
  - fully rational with regard to choice of options

39 ☐ Expected Utility Theory

- Seek to maximize positive utility (pleasure)
- Seek to minimize negative utility (pain)
- Components:
  - subjective utility: based on individual's judged weightings of utility
  - subjective probability: based on individual's judged weightings of probability

40 ☐ Which job should I take?

Company A: 50% chance of a 20% salary increase the first year

Company B: 90% chance of getting a 10% salary increase the first year

1 **Classical Decision Theory**

- Calculate *expected value* for each option:
  - Company A: .5 x .2 = .10
  - Company B: .9 x .1 = .09
- Perform similar calculations for other factors (e.g., health insurance, severance package, vacation allowance, job satisfaction)
- Assuming other things equal, choose job with Company A

2 • **Expected Utility Theory**

- Assign individual subjective weighting to various factors (salary, health insurance, etc.)
- Assign individual subjective weights to various probabilities of obtaining positive utility (strategy important)
- Calculate: Σ[p(pos)] - Σ [p(neg)]
- Choose Company based on the sum of expected positives - negatives

41 Clinical Applications of Utility Theory

– Time Trade-Off Techniques

- "Imagine that you are told that you have 10 years left to live. In connection with this you are also told that you can choose to live these 10 years in your current health state or that you can choose to give up some life years to live for a shorter period in full health. Indicate with a cross on the line the number of years in full health that you think is of equal value to 10 years in your current health state" If the person puts the line on 4, the TTO is .4
- Patient presented with iterative choices until s/he is indifferent to the choice; e.g., 20 blindness v. 5 perfect health, v. 10 perfect health, etc. If the below choice is the indifference point, the health utility of one-eye blindness is 17/20 = .85

42 Clinical Applications of Utility Theory (cont'd)

- Standard Gamble Technique
  - Patient ranks health care states along a continuum, and then is asked to make a choice like the one below; relative size of the "death" region (i.e., risk) is iteratively changed until person is indifferent to choice

43 Prospect Theory

(Kahneman & Tversky)

- Describes how individuals evaluation losses and gains – generally, we are "loss averse"
- Two stages
  - Stage I: Editing: outcomes ordered following some heuristic; set a reference point
  - Stage II: Evaluation: compute utility value and choose
- Explains a variety of economic behaviors
  - Status quo effect – insurance example (23% NJ, 53% PA)
  - Endowment effect – coffee cup examples
  - Sunk cost effect – vacation example

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45 Prospect Theory – Example Applications

- Understanding white collar crime: covering up minor crimes (failure to cut losses)
- Iraq war, other examples of organizational inertia– sunk cost effect?
- Stock investing - Why do so many financial investors hold onto a stock that has plummeted far more frequently than they keep a stock that has risen sharply, or that has maintained a steady price?
- Health decision-making: risk-taking in bad situations (e.g. HIV/AIDS)

46 Framing (Prospect Theory)

- 600 people are at risk of dying of a particular disease. Vaccine A could save 200 of these lives. For Vaccine B, there is a .33 likelihood that all 600 people would be saved, but a .66 likelihood that all 600 people will die. Would you choose A or B? (most choose A)
- 600 people are at risk of dying of a particular disease. If Vaccine C is used, 400 of these people will die. If Vaccine D is used, there is a .33 likelihood that no one will die, but a .66 likelihood that all 600 people will die. Would you choose C or D? (most choose D)

47 Anchoring and Framing Effects

- Anchoring effect (ans = 40,320)
  - Estimate: 8x7x6x5x4x3x2x1 (2,250)

  - Estimate: 1x2x3x4x5x6x7x8 (512)
- Framing effects
  - the way that options are presented affects option selection
    - risk aversion when presented with a gain options (pick small but certain gain over large but uncertain one)
    - risk seeking when presented with potential losses (choose large, uncertain loss rather than smaller, certain loss)

48 Satisficing (Simon)

- Reaching "acceptable" goals
- Notion of "bounded rationality": rationality, but within limits
- Do not consider total range of options, but consider options one by one until one meets our minimum standards of acceptability
- Probably don't reach optimal solution, but also don't spend eternity searching for one (e.g., selecting a graduate school, selecting crackers)

49 Elimination by Aspects (Tversky)

- Consider one aspect (attribute) of available options
- Form minimum criterion for that aspect
- Eliminate all options that don't conform to minimum criterion
- Then select a second aspect...and so on

50 Models of Probability Judgments

- **Descriptive:** how people reach decisions (naturalistic observation)
- **Normative:** how a decision should be made using unlimited resources.
  - Bayes' Theorem:

$$P(H|E) = \frac{P(E|H) \times P(H)}{P(E|H) \times P(H) + P(E|\text{not } H) \times P(\text{not } H)}$$

$$= \frac{(.9 \times .05)}{(.9 \times .05)+(.1 \times .95)}$$

$$= .32$$

51 Bayes Theorem Applied

- In previous example, two types of probabilities exist:
  - **"prior probability":** probability that event will occur given similar prior circumstances (e.g., p = .05 that your friend will invite your ex-husband to the party)
  - **"conditional probability":** probability that new information is true if a particular hypothesis is true (e.g., p = .90 that the car you see parked belongs to him)

52 Are we accurate probability calculators?

- Probably not...we're more conservative
- Edwards (1968): drawing chips, with replacement, from one of two bags with 70/30 mix of red/white chips. If first chip is red, what's the probability that the second chip will also be red? Actual p=.70 (subjects say p=.60)
- Meehl's criticisms of clinical decision-making and the clinical-actuarial debate

53 Probability Judgments

- Three candidates, A, B, and C are running for Mayor of Gainesville. In 6 separate polls, A led B five times. In 18 polls, C led B 9 times. In a comparison of A and C, who is more likely to win?
- It is known that 5% of the population is affected by rubadubitis. A new diagnostic test gives true positives of the disease 85% of the time, but has a 10% false positive rate. Bub has tested positive. What is the probability that he hs rubadubitis?

54 Common Heuristics in Probability Judgments

- **Frequency Heuristic:** making use of number of occurrence, rather than probability of occurrence
  - candidate example: C has more wins, but A has greater proportion of wins (5/6); most people choose C

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56 Common Heuristics (cont'd)

- **Representativeness Heuristic:** making choices based on how similar/representative a person or sample is, rather than relying on calculated probability
  - fail to use conjunctive rule: Linda is regarded as "representative" of a feminist, so most people choose "b"
  - fail to use baserates: rubadubitis example, estimates are around .85 (actual answer is .31)

57 Common Heuristics (cont'd)

- **Availability Heuristic:** using most salient, or apparent answer to guide judgment
  - Which is more likely: death by tornado or death by asthma? (asthma)
  - Is the letter "k" more likely to occur in the first or third position in English words? (3rd)
- Conclusion: people aren't very good at calculating probabilities; they rely on heuristics

58 Heuristics and Biases (Kahneman & Tversky)

- People commonly use short-cuts (heuristics)
- Heuristics lighten cognitive load, but lead to greater biases and errors
- Example heuristics:
  - REPRESENTATIVENESS: how representative instance is of universe
  - AVAILABILITY: how easily instances are called to mind

59 Examples

- All families having exactly 6 children in Pleasantville were surveyed. In 72 families, the exact birth order was GBGBBG. What is your estimate of the number of families in which the birth order was BGBBBB?
- What percentage of men in a health survey have had one or more heart attacks? What percentage of surveyed men both are over 55 and have had one or more heart attacks? (conjunction fallacy)

60 Part II: Reasoning

61 Truth Tables and Logical Operators

- Concept of propositional calculus (assertion that is either true or false)
- Limited number of operators: not, and, or, if…then, if and only if
- Truth tables chart truth value of proposition by laying out state-of-world possibilities
- Use of conditional logic

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63 Forms of Conditional Reasoning, based on "If P then Q"

- Valid Forms
  - Modus Ponens: P, ∴ Q
  - Modus Tollens: not Q, ∴ not P
- Invalid Forms
  - Affirming the Consequent: Q, ∴ P
  - Denying the Antecedent: not P, ∴ not Q
- Additional or alternative antecedents affect the use of inferential forms

64 Theories of Reasoning

- Abstract-Rule Theories: reasoning proceeds much like logical proofs
- Domain-Specific-Rule Theories: reasoning based on schematic rules specific to the type of problem (Wason's selection task)
- Model Theories: reasoning proceeds using mental models of the world (syllogisms)
- Bias Accounts: reasoning as a product of nonlogical tendencies (believability bias)

65 Abstract-Rule Theory

- Natural language premises (If A, then B) encoded by a comprehension mechanism; this mechanism is normally rational but can be derailed
- Representation of premises is related to elementary, abstract reasoning rules (e.g., modus ponens)
- If these rules do not produce conclusion, then non-logical processes are invoked
- Types of errors
  - comprehension: premise misconstrued
  - heuristic inadequacy: poor strategy
  - processing: attentional, working memory lapses

66 Abstract-Rule Account of Invalid Inferences

- Premises are re- or mis-interpreted
- Importance of "co-operative principle" (speaker tells hearer exactly what they think the hearer should know); hearer then makes invalid inferences
  - e.g.: the only way Alicia can get wet is if it rains on her

67 Status of Abstract-Rule Theory

- Can account for rule-based inference problems and for effects of alternative and multiple antecedents
- Comprehension component underspecified
- Applicable only to propositional reasoning situations

68 Domain-Specific Knowledge and Reasoning

- Posit types of situation-specific rules that are used to solve reasoning problems (probabilistically based):
  - specific prior experience
  - schemata for different types of situations (e.g., permissions, obligations)
- Rules have specific form that can be applied in all situations corresponding to that schema

69 Model Theory

- Three processes:
  - ***comprehension of premises:*** semantics and analogy
  - ***combining/description:*** models of simple premises are combined to form integrated model
  - ***validation:*** search for counterexamples or alternative models disconfirming the conclusion
- Models consume processing resources
- Errors arise from inadequate models

70 Rule v. Model Theory – an example

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- Problem 1
  - **A is to the right of B**
  - **C is to the left of B**
  - **D is in front of C**
  - **E is in front of A**
- What is the relation between D and E?

- Model: C B A
  D E

Conclusion: "D is to the left of E" (70% accurate)

2

- Problem 2
  - **B is to the right of A**
  - **C is to the left of B**
  - **D is in front of C**
  - **E is in front of B**
- What is the relation between D and E?
- Model 1: C A B
  D E

- Model 2: A C B
  D E

Conclusions (from both models: "D is to the left of E" (46% accurate)

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72 Model Theory (cont'd)

- Valid Inferences
  - develop and "flesh out" models based on propositions
  - working models out may take up processing resources
- Invalid Inferences
  - incorrect initial models (e.g., confusing biconditional with conditional)
  - can account for context effects; additionals serve as counterexamples

73 Bias Theory